



Review

Modern topics and challenges in dynamic fracture

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Abstract

The field of dynamic fracture has been enlivened over the last 5 years or so by a series of remarkable accomplishments in different fields—earthquake science, atomistic (classical and quantum) simulations, novel laboratory experiments, materials modeling, and continuum mechanics. Important concepts either discovered for the first time or elaborated in new ways reveal wider significance. Here the separate streams of the literature of this progress are reviewed comparatively to highlight commonality and contrasts in the mechanics and physics.

Much of the value of the new work resides in the new questions it has raised, which suggests profitable areas for research in the next few years and beyond. From the viewpoint of fundamental science, excitement is greatest in the struggle to probe the character of dynamic fracture at the atomic scale, using Newtonian or quantum mechanics as appropriate (a qualifier to be debated!). But lively interest is also directed towards modeling and experimentation at macroscales, including the geological, where the science of fracture is pulled at once by fundamental issues, such as the curious effects of friction, and the structural, where dynamic effects are essential to proper design or certification and even in manufacture.
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Contents

- 1. Introduction566
- 2. Dynamic fracture at the geological scale567
- 3. Dynamic fracture at the atomic scale.569
- 4. Dynamic fracture at the microscopic and macroscopic scales571
 - 4.1. Fracture roughness, crack front waves, and dissipation mechanisms572
 - 4.2. Initiation and propagation criteria.573
 - 4.3. Cohesive element modeling—links to atomistics575
- 5. Experiments577
 - 5.1. Experimental methods577
 - 5.2. Challenges for experimenters579
- 6. Intersonic and supersonic fracture580
- 7. Friction582
- 8. Problems in materials design and engineering certification.584
- 9. Dynamic fracture for manufacture586
- 10. The future.587
- Acknowledgments589
- References589

1. Introduction

At some scale, all fracture is dynamic. Our common experience in the macroscopic world may lead to the generalization that the dynamic nature of fracture is manifest only when the inertia of relatively large pieces of material is large enough that the correct balancing of the energy of fracture requires including kinetic energy. For a crack that is already propagating (beyond initiation), we may say that dynamic (inertial) effects are important when the crack tip speed is small compared to the stress wave velocities. But even when these conditions for dynamic effects at the macroscopic scale are not met, still at the atomic scale, the most fundamental for understanding crack propagation, the inertia of individual atoms must be accounted for in depicting their separation from one another, even when the crack at the macroscopic scale appears to be advancing quasi-statically. Bond rupture is a dynamic process. The dynamic fracture problem is the most fundamental in the science of fracture.

The dynamic fracture literature has been sustained in a wide number of journals by several distinct scientific communities, which have been focused on different objectives. The communities have been divided partly by the scale of the systems they consider, which ranges from atoms to earthquake fault lines, partly by the phenomena they have chosen to address, and partly by the balance of their leanings towards curiosity-driven science or technology. Some cross-referencing among the different bodies of work is found, but not as much as the inter-relationship between the ideas being explored would justify. Here progress is reviewed in sections whose titles reflect the major boundaries between the dynamic fracture communities, because this is a convenient categorization, but fixed points where analogous results or concepts appear in different sections, rescaled or otherwise restated, are repeatedly in evidence. A rich prospect presents itself for transferring concepts and methods from one field to another.

Progress in curiosity-driven science has been rapid and fascinating. But the technological relevance of dynamic fracture to structures and systems that will sustain impact or ballistic threats also remains very high. The insertion of the new scientific and engineering concepts and methods into engineering design is therefore a very timely topic for new research.

This review is intended to be neither a complete survey of all areas of dynamic fracture nor historically complete in any one area. Instead, particular themes are developed, which were identified as especially promising focuses for new research by the group that participated in the Ringberg Workshop on Dynamic Fracture (Bavaria, July, 2003). Topics that are not strongly referenced include fragmentation; the dynamic behaviour of comminuted material, e.g., ahead of a penetrator; applied work on ballistic structures; dynamically loaded stationary cracks in layered materials; dynamic fracture in piezoelectric materials; and dynamic dislocation emission from crack tips.

2. Dynamic fracture at the geological scale

Earthquake modeling is concerned with dynamic shear cracks under slow loading. A stream of recent work by Rice and his colleagues has led to new understanding of the complex phenomena of dynamic cracking under such conditions. First, [Ben-Zion and Rice \(1997\)](#) developed a dynamic simulation scheme of dynamic shear cracks under slow loading, which was further improved by Lapusta and colleagues ([Lapusta et al., 2000](#)). [Lapusta and Rice \(2003\)](#) went on to study nucleation of shear cracks and their early dynamic stages in the context of dynamic shear ruptures. They established that nucleation processes of large and small events are very similar. Further, events at different scales interact: the irregular initial dynamic phases of large events are influenced by stress concentrations induced by the initiation and arrest of small events.

Self-healing or pulse-like dynamic shear ruptures are also being actively investigated in the field. In a pioneering paper, [Heaton \(1990\)](#) showed that some earthquake observations can be explained if the duration of the dynamic sliding at

each point along the interface is short compared to the overall time of the dynamic event. Since then many similar observations have been made. This mode of rupture propagation was called self-healing or pulse-like, in contrast to the “crack-like” mode in which the duration of sliding at a point is comparable to the overall duration of the dynamic event. In parallel theoretical work, [Zheng and Rice \(1998\)](#) demonstrated conditions under which crack-like rupture modes are indeed replaced by pulse-like modes at velocity-weakening interfaces. [Adams \(1998\)](#) further demonstrated that the dynamic coupling in the frictional relationship between normal stress and shear displacement discontinuity (slip) leads to pulse-like slip on a generic bimaterial interface. [Andrews and Ben-Zion \(1997\)](#) were the first to simulate a slip pulse on a dissimilar material interface with a constant coefficient of friction. They observed splitting of the pulse which was later shown, in a fundamental revelation concerning the nature of friction, to be the consequence of ill-posedness of the problem when formulated with constant friction.

[Ranjith and Rice \(2001\)](#) took up this problem using stability analysis to demonstrate a wide range of parameters for which the problem of dynamic sliding with a constant coefficient of friction is ill-posed. For Coulomb friction, ill-posedness is always found when a generalized Rayleigh wave can propagate along the interface (a condition on the degree of elastic dissimilarity of the materials); and above a critical value of the friction coefficient when the generalized Rayleigh wave does not exist. Ill-posedness is directly related to the physics of friction: [Ranjith and Rice](#) showed that ill-posed problems can be regularized by substituting a friction law in which the shear strength in response to an abrupt change in normal stress evolves continuously, rather than abruptly, with time (or equivalently with slip distance) toward the corresponding Coulomb strength. Such a law has independent experimental justification ([Prakash and Clifton, 1993](#)); but an interesting question is whether it should be necessary that a friction law must be of such a kind as to lead to a well-posed problem in elastic wave analysis. [Cochard and Rice \(2000\)](#) reinforced the validity of the analytical results of [Ranjith and Rice](#) by numerical simulations of dynamic slip on dissimilar material interfaces.

This important line of research has shown that rupture along a bi-material interface has remarkable dynamic properties that may be relevant to many problems, over broad ranges of scales, from geophysics to composite materials. The problem has the fascinating feature that it starts in a situation well-posed as a static continuum problem but evolves dynamically to a non-continuum state of slip pulses. Recent papers on this topic include, e.g., [Ben-Zion \(2001\)](#), [Ben-Zion and Huang \(2002\)](#), and [Ranjith and Rice \(2001\)](#).

The role of friction in shear rupture is very closely related to the geometry and connectedness of the rupture surface or surfaces. Significant recent work has probed how the character of earthquake rupture should be influenced by slip weakening friction laws ([Peyrat et al., 2001](#)), non-planar and branched fault structures ([Aochi and Fukuyama, 2002](#); [Aochi et al., 2002](#)), and disconnected faults ([Harris and Day, 1993](#); [Harris and Day, 1999](#)). Analogies to all these phenomena arise at different scales in structural materials.

The effects of nonlinear rheology that extends beyond the elastic regime, which create some interesting and important characteristics in studies of general dynamic cracks (see below), are also beginning to attract attention in earthquake studies. The works of Gao (1996, 1997) on hyperelasticity and Geubelle on damage cover some general aspects of this broad topic (Maiti and Geubelle, 2002; Maiti et al., 2004). Related studies on the evolution of earthquakes and faults can be found in Lyakhovskiy et al. (1997, 2001) and Ben-Zion and Lyakhovskiy (2002).

3. Dynamic fracture at the atomic scale

A broad development that many participants of the Ringberg Workshop found particularly inspiring and at many moments simply beautiful is the bridge that has grown between continuum and atomic (or other discrete) descriptions of dynamic fracture. As typified by the work of Abraham et al. (2002, 1994), the rapid advance of computer power during the last few decades has revolutionized the field. In 1965, a typical problem addressed by computational physicists consisted of about 100 atoms. By the turn of 21st century, the system size had reached 1 billion atoms, roughly corresponding to a cubic crystal of 250 nm in edge length. This size is certainly in the regime where a “hand shake” to continuum formulations becomes possible. Spectacular examples are now available (Abraham et al., 2002) of 1-billion-atom simulations of work hardening and supersonic fracture. Projections assuming the continuation of Moore’s law predict that by 2007 the state of the art will have reached a trillion atoms and simulations on the billion-atom level will become routine.

Atomistic studies, largely represented by molecular dynamics simulations, have been used significantly to date to provide fundamental understanding of underlying basic physical processes of dynamic fracture, rather than being predictive or specific to a particular material. Examples are the method of lattice dynamics modeling, which originated with Sleypan and is advocated by Marder and Kessler (Marder and Gross, 1995; Sleypan, 1981); or Abraham’s studies of simple model materials. While Abraham et al. (1994) opened up the possibility of studying fracture through large-scale simulations of model materials, the question remains (especially in the minds of experimentalists!) of how much applicability such results have. This is particularly an issue given the very complex microstructure on different length scales in real materials and—in contrast—the often very simple and ideal microstructure in simulations. Atomistic work also often suffers from the strict limit to the total time that can be modeled. This often leads to calculations being carried out at very high loading rates (perhaps even 10^5 times the rate of practical interest); and it is not at all clear that fracture processes at artificially high rates and therefore high stresses are representative of what occurs at lower rates. More studies are obviously needed to provide a more seamless bridge between atomistic approaches and continuum-scale theory and experimentation (rather than “isolated” results that are possibly only valid in very specific cases). This is a challenging task given the fact that fracture is a very complex process.

However, there are some promising areas where the atomistic viewpoint has been successfully coupled to continuum-scale theory or experiments. An example of such “handshaking” of experiment-theory-simulation is the intersonic fracture of mode II cracks, where experiments by Rosakis et al. (1999) have stimulated the recent MD simulations of Abraham and Gao (2000). The MD simulations were able to match essential features of intersonic fracture that are observed in the experiments and also predicted by cohesive modeling (Needleman, 1999) and continuum elasticity (Gao et al., 2001).

Atomistic studies are often associated exclusively with molecular dynamics simulations. An alternative class of analytical methods developed by Slepyan (1981, 2002), using a formulation where atoms remain associated with particular points on a lattice, can also, and in much smaller computations, reveal insight into atomic-scale dynamic fracture. For example, Marder and Gross (1995) made use of Slepyan’s technique to calculate a velocity gap and transverse instability during dynamic crack propagation. Hauch et al. (1999) attempted to compare theory and experiment for brittle fracture without phenomenological parameters. Starting from the atomic point of view, Gao et al. (2001) demonstrated existence of intersonic cracks under shear dominated conditions, while Gerde and Marder (2001) demonstrated the existence of self-healing cracks under mixed-mode conditions.

Discussion touched with a hint of passion made clear during the Ringberg Workshop that there remain some fundamental open questions regarding atomistic modeling. How predictive are atomistic simulations of dynamic fracture? How can we develop tools that use atomistics in some way to model very complex fracture processes such as the collapse of a building? The view that atomistic simulations could eventually be developed into such practical tools for engineers was received as rather radical, yet not impossible. One thing that can be safely said is that atomistic modeling will play an increasingly important role in the next decades along with the rapid advance of computer power. Molecular dynamics is potentially an important part of multi-scale methods describing very complex processes. An important issue is the development of multiscale methods such as continuum methods which incorporate atomistic information. Examples are the VIB method by Gao, Klein, Huang and co-workers (Gao and Klein, 1998; Klein et al., 2001; Nguyen et al., 2004; Zhang et al., 2002b) and the concurrent multiscale methods where some regions are treated by finite elements while others are treated by molecular dynamics, such as the FEAt method (Kohlhoff et al., 1991), quasi-continuum method (Shenoy et al., 1998) and the MAAD method (Broughton et al., 1999). A method that aims between the quantum and classical formulations has been suggested by De Vita and Car (1998), who dynamically fit empirical potentials in the region of interest to forces calculated from quantum methods. The computational approach includes refitting the forces after a short time interval to guarantee accurate match with the *ab initio* forces at particular epochs.

In some cases, model potentials and classical molecular dynamics fail to explain the observed dynamic fracture phenomena and quantum mechanics modeling appears to be required. Gumbsch and colleagues have illustrated this point for cleavage fracture in silicon. Pérez and Gumbsch (2000a, b) showed that anisotropy in the preferred cleavage direction in silicon, which is experimentally well documented,

can be explained only on the basis of quantum mechanical (density functional theory or DFT) calculations. In these simulations, quantum calculations of the electronic structure are performed at each load increment for the current positions of the atoms (nuclei) in the model. The forces (energy derivatives) acting on each atom are then deduced from the computed electronic structure. Gumbsch (2001) discusses some general aspects of atomistic modeling of fracture including bond breaking, lattice and bond trapping, and relevant energies, and emphasizes the importance of realistic descriptions of electronic structure. Discovering how to reproduce quantum mechanical results with simpler atomistic descriptions remains a challenge. So far the quantum mechanical (DFT) results for the bond breaking process in silicon and the existence of directional anisotropy can only be reproduced with advanced tight binding methods (Gumbsch, 2001) and not with simple potentials in a classical MD simulation. Thus the warning is laid down: classical mechanics may be inherently inadequate for understanding the most fundamental aspects of fracture. However, unanswered questions abound in this fascinating new direction in modeling. For example, the pseudopotential approximation used in the quantum calculations of Gumbsch and colleagues precludes relaxation of core electron states. The validity of this approximation when atomic separations undergo the large changes characteristic of fracture needs to be assessed. A further fundamental question is whether quantum effects in the vibrational excitations of atoms, which are absent in all models to date, since atomic motion is treated as Newtonian, might enter into fracture for some temperatures or for species with low atomic masses.

Other experimental studies on single crystal silicon partly confirm theoretical ideas and partly confute modelers by pointing out further complexity (Cramer et al., 2000). Thus, addressing the existence of a velocity gap in silicon, Be'ery et al. (2003) recently produced evidence of cracks propagating at very low velocities under controlled conditions in silicon. But further work from the same group discovered new mechanisms for the deflection of a crack from one cleavage system to another as a function of crack velocity and crystallographic orientation (Sherman and Be'ery, 2004). Shilo and colleagues demonstrated, both experimentally and by modeling, the potentially important interaction of a crack in silicon with microstructural defects (dislocations) and ramifications for the stability of the crack front (Shilo et al., 2002).

Further work from the group of Gumbsch (Rudhart et al., 2003) has addressed dynamic fracture in quasi-crystals, a class of materials structurally between glass and crystal, and has found distinct changes in fracture mode with temperature. The fracture modes in quasi-crystals include brittle fracture, brittle fracture following “virtual” dislocation emission, and, at high temperature, glass-like breaking of bonds in front of the actual crack tip, and crack growth by linking of the resulting microcracks.

4. Dynamic fracture at the microscopic and macroscopic scales

Dynamic fracture may involve various mechanisms of nonlinear damage, many of which act at the microscopic scale: granulation, void growth and coalescence in the

failure of many ductile solids, phase changes, crazing, and the deformation and evolution of junctions between frictional contact asperities in problems of slip failure. Many aspects of these phenomena can be addressed as problems in classical mechanics. Where the phenomena can be represented by (generally nonlinear) constitutive laws, their import for dynamic fracture can often be understood by analysing an appropriately defined problem in macroscopic fracture. Recent advances in understanding the bridge between microscopic and macroscopic aspects of dynamic fracture are as follows.

4.1. Fracture roughness, crack front waves, and dissipation mechanisms

Measurements and modeling of fracture surface roughness in materials of different type, including ductile alloys and brittle glasses, and under dynamic and static loading, has led to new insight into the conditions for crack propagation. First, remarkably similar self-affine fracture surface morphology has been observed in fatigue fractures in metals and stress corrosion cracks in glasses (Daguier et al., 1997), a result that has stimulated a search for mechanisms that could be universal to such disparate cases. A further clue on the trail came from molecular dynamics simulations, which showed that similar morphology can also arise in the fracture of glass in the dynamic regime (Nakano et al., 1995). Other simulations revealed that a dynamic crack in amorphous glass propagates by the nucleation and coalescence of cavities, much as a crack in a ductile metal (Celarie et al., 2003; van Brutzel et al., 2002). Thus apparently similar roughness is found in static, stress corrosion, and dynamic cases; and theory hints at universality among very different material classes. A separate and rather provocative theoretical study has further suggested that *only* dynamic conditions near the crack tip can account for the roughness; dynamic stress transfer affects the crack-tip conditions significantly even in cracks that are propagating quasi-statically from the macroscopic view (Ramanathan et al., 1997).

Simultaneously with these developments, basic mathematical tools were developed by Willis and colleagues for analysing in-plane and out-of-plane perturbations of a propagating crack front and dynamic stress transfer during crack propagation (Willis and Movchan, 1995; Willis and Movchan, 1997). These techniques have found immediate application in understanding various features of earthquake rupture and they have been used, in particular, to study crack front waves for generic dynamic cracks (Morrissey and Rice, 1998; Ramanathan and Fisher, 1997). In the latter works, a localized perturbation of a propagating crack front has been shown to be able to propagate without decay along the crack front, resulting in a new kind of wave, called a crack front wave, which is a distinct mode from the classical longitudinal, shear, and Rayleigh wave modes. The association was then suggested, that crack front waves may be responsible for the small-scale, self-affine fracture surface roughness found experimentally and in simulations for so many different materials in both dynamic and (macroscopically) static loading (Bouchaud et al., 2002). However, crack front wave theory does not predict the most commonly observed exponent of scale dependence (0.8), whereas the competing idea that the

roughness originates in multiple voiding and void coalescence ahead of the main crack does suggest the correct exponent (Bouchaud et al., 2002).

Attempts have been made at direct experimental verification of crack front waves, but the outcome remains subject to some controversy. Fineberg and co-workers reported observations of solitary waves, which they inferred to be crack front waves (Sharon et al., 2001). However, Bonamy and Ravi-Chandar (2003) have claimed contrary experimental evidence, which seems to indicate that the solitary waves observed in the experiments of Fineberg may be due to an interaction between shear waves and the propagating crack front. More experiments are obviously needed to settle this fundamental point.

Fracture surface roughness may also be the result of microcracking, which is an important dissipation mechanism for a broad class of materials (Ravi-Chandar and Yang, 1997; Sharon et al., 1996; Washabaugh and Knauss, 1994). Microcracks can roughen cracks by acting as nuclei for macroscopic crack branching; and they can lead by this and other mechanisms to shielding and amplification effects. When numerous microcracks appear as a cloud, they reduce the effective stiffness of the material surrounding the macroscopic crack tip so that the effective wave speed and therefore the crack speed through the damaged material are reduced (Gao, 1996) and the macroscopic fracture toughness increases. Early studies of the effect of diffuse microcracks used spatial averaging to develop a phenomenological cohesive law that reduced the nonlinear material to a narrow band or line of displacement discontinuity. The evolution of the damage zone was modeled (Johnson, 1992; Yang and Ravi-Chandar, 1996). More advanced studies by Gross and colleagues represent the microcracks individually, using a time domain boundary element method, which is particularly efficient for multiple crack problems in homogeneous materials, i.e., when heterogeneous elasticity is unimportant. Crack closure effects have been taken into account and crack branching and the interaction between macro- and microcracks during crack growth have been investigated in detail (Rafiee et al., 2003, 2004; Seelig and Gross, 1997, 1999). These studies show that microcracking can explain the increase of fracture toughness with crack velocity and also predict an upper bound for the crack velocity, which lies far below the Rayleigh wave speed. These results agree well with theoretical and experimental findings of other authors (Hawong and Kobayashi, 1987; Ramulu and Kobayashi, 1985; Shukla et al., 1990). Since boundary element methods facilitate the analysis of arbitrary crack paths and the formation of microcracks, crack branching, crack closure and friction, they are being adopted and further developed as an appropriate tool for other macroscopic problems, including problems at the geophysical scale (Fedelinsky and Aliabadi, 1997; Tada and Yamashita, 1997).

4.2. Initiation and propagation criteria

The question of the correct way of stating the condition for static crack propagation has lain unresolved at the heart of static fracture mechanics since its inception. In many engineering applications, especially involving single cracks, conditions based on energy release rate or critical stress intensity factors have

enjoyed great practical value. However, cases exist in quasi-static loading where an energy condition clearly predicts the wrong crack path; and cases are ubiquitous where a dominant stress singularity is a fundamentally unrealistic view of crack-tip conditions. Now, from the work on crack front waves and its possible dominance of fracture surface roughness, it would appear that the ultimate answer to the quasi-static fracture criterion may not be approachable by calculations of static behaviour alone but may lie in essentially dynamic phenomena.

An intriguing problem related to shear loaded cracks is the so-called failure mode transition first identified in experimental work by [Kalthoff \(1988\)](#). Depending on the impact velocity, a shear-loaded crack may propagate either under local opening (mode I) conditions, by deflecting through a kink angle; or in mode II along its initial direction, by forming an adiabatic shear band. This transition, which has been observed for a variety of metallic alloys ([Zhou et al., 1996](#)) and commercial polycarbonate ([Ravi-Chandar, 1995](#); [Rittel, 1998a](#)) implies velocity dependence in the failure criterion. One criterion proposed is a simple rate-dependent critical mode II fracture toughness ([Kalthoff, 2003](#)). But one would guess that the mode II toughness is likely to be affected by friction, which will be affected by fracture surface roughness; and so a possible connection to work on crack front stability and crack front waves is completed.

In an early, seminal work on initiation, using deeply notched, thin sheets of Homalite loaded in mode I by a clever electromagnetic method, [Ravi-Chandar and Knauss \(1984\)](#) showed strong correlation between the onset of crack propagation and the crack-tip stress intensity factor. Thus for engineering applications, the idea of a dynamic initiation toughness arises, in analogy to the fracture toughness used in quasi-static fracture, where the critical toughness might be a material constant that could be measured in a simple standard experiment. However, the definition of a standard experiment to measure dynamic toughness has not been straightforward. [Rittel et al. \(1992\)](#) have proposed and developed some simple methods, but a crucial difficulty arises in determining the time of initiation. This issue relates to the three-dimensional nature of the fracture process: all measurements probe only the surfaces of the specimen, so that the moment of advance of a submerged crack tip, and thus the stress state at the crack tip at the critical instant, is difficult to know. A second challenge arises: while ample evidence shows that initiation toughness depends on the loading rate, no clear relationship exists between this trend and material type, so that predictive capability is still limited. The resolution of this doubt must lie in experiments that reveal fracture micromechanisms operating in each specific material. Some recent work indicates the power of such evidence in understanding engineering initiation data. For example, commercial polymethylmethacrylate (plexiglass) shows a large increase in initiation toughness with the loading rate ([Rittel and Maigre, 1996](#)), which is associated with a marked increase of the roughness of the fracture surface next to the fatigue pre-crack front. This suggests a transition in damage mechanism involving the possible creation of multiple microcracks under the passage of the first stress wave, to an extent that depends on loading rate. Further supporting this idea, numerous non-propagating microcracks have been found in dynamically loaded cermets (porous ceramic matrix

infiltrated with a metallic binder), but are absent in statically loaded material of markedly lower initiation toughness (Kaplan et al., 2004; Rittel et al., 2004). But the details of how microcracks interact with the main crack in the stage of damage development that can be identified with engineering initiation remain unresolved.

4.3. Cohesive element modeling—links to atomistics

Perhaps the most important development in computational fracture mechanics in the last decade has been the introduction and refinement of the cohesive element methods (Camacho and Ortiz, 1996; Xu and Needleman, 1994). Cohesive elements introduce the possibility of tractions surviving across fracture surfaces after the propagation of a crack and, furthermore, allow the crack to follow any path during a simulation, rather than being confined to a pre-determined path. Some very appealing simulations have recently appeared of multiple crack development and other complex fracture habits, which could not be realized using prior methods in which the fracture paths had to be specified a priori (Camacho and Ortiz, 1996; Klein et al., 2001; Xu and Needleman, 1994).

However, the newness of cohesive surface methods is evident in the fact that some intrinsic difficulties remain with the original formulations (Klein et al., 2001). The problems include artificial softening of material properties as the size of cohesive elements decreases; and mesh dependence in the direction of crack branching (at least at the first onset of a branch). Similar problems have been observed by Falk et al. (2001).

In response to these difficulties, a new class of cohesive elements, which exploit the partition of unity property of finite elements, has been developed, which relieve constraints on the admissible directions of new fracture surfaces by allowing cracks to propagate on any surface within an element, rather than only along element boundaries (Belytschko et al., 2003; Moës et al., 1999; Zi and Belytschko, 2003). Beyond the curious historical fact that these new cohesive element models originated in dynamic fracture research, one can easily envisage their widespread use in many other fields (Remmers et al., 2003). Belytschko also proposes an interesting propagation (or initiation) criterion, which is the loss of hyperbolicity in the wave equations in any region of material (Belytschko et al., 2003). This criterion appeals both for its physical sense and its convenience in numerical work. New elements and propagation conditions that do not artificially favour any direction may lead to significantly improved cohesive method simulations.

Cohesive forces and related length scales also have prominent roles in the dynamic fragmentation of a granular material. The phenomenon of fragmentation during dynamic wave propagation has received some attention by approximate analytical methods. In the simple case of fragmentation of a brittle homogeneous medium subject to uniaxial stress waves, Drugan derived a characteristic length, l_{\min} , for the minimum elastic fragment size, which is related to the wave speed and certain features of a cohesive law that is similar to that used in the first cohesive element finite element methods (Drugan, 2001). The length scale can be rewritten, within a factor of order unity, as $l_{\min} = \delta^* E / 2\sigma_{\max}$, where δ^* and σ_{\max} are the critical

displacement and stress in the cohesive law and E is a reduced modulus. This result is similar in form to the characteristic bridging length scale, l_{ch} , introduced by Hillerborg, Rice and others to characterize the length of the bridging zone in a bridged static crack (Cox and Marshall, 1994; Hillerborg et al., 1976; Massabò and Cox, 1999; Rice, 1980). However, Drugan's length scale is independent of the shape of the cohesive law for displacements beyond the critical displacement. The length scale, l_{ch} , is more often written in bridged crack work as $l_{\text{ch}} = GE/\sigma_{\text{max}}^2$, where G is the fracture energy (total area under the cohesive law), and only takes the form $l_{\text{ch}} = \delta^*E/2\sigma_{\text{max}}$ in the special case of a Dugdale law (rectangular cohesive law, $G = \sigma_{\text{max}}\delta^*$). The difference in the length scales reflects a distinction in the physics of fragmentation and crack propagation. In the absence of a pre-existing flaw, fragmentation is controlled by the onset of mechanical instability: when crack planes separate beyond the peak of the cohesive law, unstable further separation is possible (with due modification to account for the wavelike time variations of stress fields). Thus the characteristics of the peak of the law are the only characteristics referenced. Crack propagation is controlled, in contrast, by energy; and the work of fracture includes the entire area under the cohesive law. Researchers analysing the spallation failure of thin film interfaces under shock loading have generally taken the view that separation occurs without crack propagation (and is therefore a measurement of strength rather than work of fracture) (Gupta et al., 1994). Other materials problems, in which crack propagation and fragmentation are present simultaneously, raise some interesting questions of how the different length and time scales will play out; under what conditions, e.g., combinations of loading rates and cohesive laws of different shapes, will new damage be dominated by growth from existing cracks or fragmentation of pristine material?

Drugan's length scale may also be viewed as characterizing a zone of stress relief on either side of an existing crack and has an analogue in static multiple matrix cracking seen in fibrous composites. There the characteristic length of stress relief (and thus the crack spacing) depends on the strength of the frictional coupling between the matrix and the fibres, since stress builds up again near a crack in the matrix via the frictional transfer of load from the undamaged fibres (Aveston et al., 1971). And a combination of dynamic tensile fragmentation and frictional load transfer effects might well account for an interesting prediction of multiple cracking seen in some recent simulations of dynamic shear fracture (Belytschko et al., 2003). In those simulations (based on cohesive element methods with no pre-determined fracture system), small cracks are predicted to initiate approximately normal to the plane of a dominant shear crack, with a roughly uniform separation. Their spacing might be determined by the propagation of tensile stress waves along the flanks of the shear crack. While friction was not included in the simulations, the crack spacing could also, in a real material, be modified by frictional tractions acting along the dominant crack plane (analogous to the interface friction seen in fibrous composites).

An interesting degree of universality can be discerned in the computational approaches that have evolved in continuum fracture modeling and the older discipline of atomistic studies. In some aspects, the cohesive surface modeling of

dynamic fracture can be viewed as incorporating atomic interactions along discrete surfaces. One sees in atomistic simulations rafts of atoms moving almost as elastic entities, separated by bands of severely displaced atoms. The analogy with elastic domains in a continuum simulation separated by localized damage represented as cohesive zones is very appealing.

In the future, it will be important to establish a deeper interaction between experiments and simulations performed at either the atomistic or continuum (cohesive element) scale. It should become an important objective of simulation groups to move beyond idealized potentials, so that they can better match realistic conditions and contribute to understanding specific issues in dynamic fracture. Particularly promising areas in which atomistic and cohesive element simulations could play an important role include crack front waves, dynamic crack branching, hyperelasticity, friction, and crack initiation.

5. Experiments

In spite of interesting advances in the realism of numerical simulations, the ultimate reference for newly suggested concepts in dynamic fracture remains, very firmly, experiments. The challenge of obtaining highly resolved, detailed observations of cracks in the dynamic regime continues to be among the most severe in the fracture world (perhaps only rivaled by fracture experiments at very high temperatures). Nevertheless, the dynamic fracture community has recently invented some noteworthy and clever new experiments, from which insight far beyond what was accessible by experiments 20 years ago is now being derived. In these experiments, both materials and mechanics aspects of the dynamic fracture problem have been targeted.

One can broadly distinguish two types of studies in modern experiments. The first concerns single crystals or amorphous polymers. While they have practical relevance in their own right, e.g., cleavage of silicon wafers (see below) and dynamic failure of plastic components, these materials, because of the absence of heterogeneous morphology, may also be considered model materials. The second deals with more complex bodies, either structural materials such as multi-phase alloys and composites; or natural systems, including biological materials and the earth's crust.

5.1. *Experimental methods*

Perhaps the most informative full-field experimental techniques for revealing the details of interactions between stress waves and propagating cracks are based on optical interference. While the method of caustics seems to be less used nowadays, optical interferometric techniques, such as dynamic photoelasticity and the coherent gradient sensor method (CGS) have gained increased popularity (Rosakis, 1993). The latter especially has been developed into a powerful and convenient tool for recording displacement gradients and stress fields. In an elastic material, the displacement gradient contours measured by CGS correspond to the stress

difference, $\sigma_1 - \sigma_2$, where (x_1, x_2) are the in-plane coordinates. Early work used photographic film and rotating mirrors, but now similarly high-density information can be recorded with film replaced by programmable high-speed digital cameras, with current systems yielding a total of about 48 frames at rates in excess of $2.5 \times 10^6 \text{ s}^{-1}$ (e.g., Coker and Rosakis, 2001).

A major advance in displacement measurement has also occurred over the last 10 or so years in earthquake studies, through the advent of synthetic aperture radar and the global positioning system. Exploiting the positional stability of space-borne instruments, these methods have made possible displacement measurements over geological scales with unprecedented accuracy ($\sim 10 \text{ mm}$) and richness of data (Bamler and Hartl, 1998; Klees and Massonet, 1998; Massonet and Feigl, 1998). While the measurements are restricted to surface information, solution of inverse problems can yield quantitative data about the shape and angle of the slip plane; the state of bulk elastic strain near the slip plane; and estimates of the frequency of slip events (e.g., Mayer and Lu, 2001; Price and Sandwell, 1998; Xia et al., 2003; Zhang et al., 2002a).

Beyond imaging of displacement fields, much can be learned about dynamic fracture from thermal and optical emissions. Temperature fields contain direct information about the thermodynamics of nonlinear or inelastic processes, both at the crack tip and in the crack wake, especially, in the latter region, friction due to sliding crack contact. While infra-red emissions from dynamic cracks were first detected some decades ago (Fuller et al., 1975; Weichert and Schoenert, 1974, 1978a, b), high-speed infrared array imaging devices have now added the enrichment of being able to capture spatially resolved images. Even though pixel resolution remains very modest, due to delays in transferring the data from the device to storage, crack tip and crack wake processes can be discriminated (Zehnder and Rosakis, 1991). Thus, e.g., direct measurements of temperature changes due to friction have been reported. Despite the limited resolution of this device, the thermal structure of an adiabatic shear band could be characterized for the first time, and as technology progresses, high-speed thermal cameras will arrive to complement existing high-speed optical cameras (Guduru et al., 2001; Zhou et al., 1996). Moreover, one may now think of coupling the high-speed cameras to appropriate microscopic lenses to reveal unprecedented information about dynamic fracture phenomena at the microscale. At present, such a direct link to microstructure is still missing for all the phases of dynamic fracture, namely initiation, propagation, and arrest. At the macroscale, inferences of temperature can be made in earthquake slip zones by examining the crystallographic phases found along the fault (Otsuki et al., 2003).

In laboratory tests, temperature measurements have occasionally led to surprising implications. For example, thermoelastic coupling effects (changes of temperature with elastic strain) have commonly been assumed to be negligible, based on experience with smooth specimens. However, recent work (Rittel, 1998b) has shown that this is not always the case: in brittle polymeric materials, the crack-tip temperature may drop significantly upon rapid loading. Such a thermoelastic effect may be important in the ductile to brittle transition and deserves additional study

and incorporation alongside plasticity effects in modeling the overall crack-tip thermodynamics.

Light emission during fracture has also been the subject of numerous investigations, ranging from ice to metal through ceramics, carried out mostly by physicists (Yasuda et al., 2002). These fascinating results remain largely isolated from the mainstream of dynamic fracture experiments. Nevertheless, the advent of sophisticated atomic-scale simulations, including quantum calculations of changes in electronic state during dynamic fracture, opens the possibility of using light emission to test models of some of the most fundamental aspects of fracture.

The quest for understanding mechanisms of dynamic fracture during earthquakes (the geological scale) has led to an exciting advance in laboratory experimental methods, with relevance to all scales. Rosakis and colleagues have demonstrated that some key features of earthquake dynamics, including intersonic rupture and rupture initiation under slip or velocity-weakening friction laws, can be captured in the laboratory using Homalite plates held together by friction forces under compression (Xia et al., 2004). This is interesting because there is an enormous difference in length scales between seismic faults and laboratory materials. A “laboratory earthquake” is initiated along a set of pre-designed fault lines (material boundaries) by an electric spark that simulates a seismogenic source. Pulsed slip ensues, similar to that in an earthquake slip event. While the simulation in a small specimen of events occurring naturally on a geological scale is eye-catching, the same experimental technique may prove equally valuable in probing dynamic friction effects in structural materials, such as laminated composites. The technique has the potential of being further developed into a unique experimental platform to test ideas and models of mode II and mixed mode cracking under various loading, environmental, and materials conditions. An example demonstrated by Rosakis is the study of dynamic branching and transfer of slip along interacting fracture planes (fault lines or material interfaces), with results comparable to seismic measurements and theoretical models developed by Rice and co-workers (Kame et al., 2003; Poliakov et al., 2002).

Such novel laboratory experiments provide bridges not only between theory and observations but also between small and large-scale phenomena. A coherent approach combining traditional seismic and fracture measurements and theoretical modeling with modern laboratory experimental tools and large-scale computer simulations may revolutionize the fields of seismology and dynamic structural design.

5.2. *Challenges for experimenters*

Major experimental challenges remain, in both fundamental and engineering aspects of dynamic fracture. One issue of continuing engineering importance is the problem of defining and characterizing a condition for crack initiation under dynamic conditions. For example, in many structures intended for long life, initiation of a single crack under a rare dynamic load, e.g., foreign object damage in an aircraft turbine blade, can be enough to reduce the fatigue life under normal duty cycling quite dramatically.

In recent work, Sharon and Fineberg (1996) produced a large body of experimental evidence related to the evolution of crack velocity and the nature of the micromechanisms that lead to crack branching. Crack branching was also investigated by Ravi-Chandar and Yang (1997), in terms of the activation of secondary microcracks near the main crack tip. Yet, looking beyond such investigations, how microstructure affects the dynamic fracture process—its effect on crack branching, initiation, etc.—remains very much an open question. This problem has old roots, having concerned pioneers in the field of quasi-static material fracture, such as the late Tetelman (Tetelman and McEvily Jr., 1967). Yet, new tools, such as high-speed optical and thermal cameras coupled to microscopes and sophisticated numerical codes will allow a re-examination of the structure–property relationship. In the modern context, some new aspects of this overarching problem challenge experimenters. For example, dynamic delamination in composites poses difficult questions regarding multiple cracking, mixed mode propagation, and the effects of long zones of friction in the crack wake (see Section 7).

Other new challenges in dynamic fracture arise in the study of natural materials. The dynamic ejection of magma that occurs during a volcanic eruption presents phenomena not encountered elsewhere. Here, one deals with the dynamic fracture of a highly viscous material, with the driving force for fracture provided by the pressure in diffusely distributed bubbles. The rate of decompression will affect the character of the damage development and thus the nature of the volcanic eruption. Ichihara and colleagues have illustrated the essential phenomena believed to be involved by a series of model laboratory experiments interpreted with fluid mechanics concepts (Ichihara et al., 2002). But dynamic fracture mechanics concepts may also be germane: the model experiments show a ductile to brittle transition in the failure mode with rising decompression rate. Additional experimental work is needed to characterize the failure micromechanisms of this complicated bubbly material and their rate sensitivity. Simultaneously, the fragmentation process might be analysed using fragmentation models in which the dynamic fracture toughness (or cohesive properties) of the material appears explicitly, thus establishing an additional link between the mechanics and physics of fracture in geomaterials.

6. Intersonic and supersonic fracture

Intersonic fracture, a concept born in theoretical work in earthquake science (Andrews, 1976; Burridge et al., 1979), has attracted much attention in the last few years, mainly due to Rosakis' experimental work (Rosakis et al., 1999). These most engaging data have motivated new cohesive element modeling (Geubelle and Kubair, 2001; Needleman and Rosakis, 1999) and molecular dynamics studies (Abraham and Gao, 2000; Gao et al., 2001). Fundamental solutions for intersonic mode II cracks have been presented (Huang and Gao, 2001) and exploited to construct transient solutions of intersonic crack propagation (Guo et al., 2003) and to study a mode II crack suddenly stopping after propagating intersonically for a short time (Huang and Gao, 2002).

A comprehensive review of the historical development of studies on intersonic cracks can be found in Rosakis (2002). In the past decade, scientists working at all length scales, from the atomistic, the continuum, all the way up to the scale of geological ruptures, have undertaken joint efforts to complete this chapter of fracture mechanics. Early contributions to the theoretical literature of dynamic subsonic and intersonic fracture highlight significant differences between tensile and shear cracks. Following inferences from earthquake data that intersonic cracking occurs in nature (Archuleta, 1984), direct laboratory observations (Rosakis et al., 1999) have provided a framework for discussing the physics of intersonic shear rupture occurring in constitutively homogeneous (isotropic and anisotropic) as well as in inhomogeneous systems, all containing preferable crack paths or faults. Experiments, models, simulations and field evidence at all length scales have been used to discuss processes such as shock wave formation, large-scale frictional contact and sliding at the rupture faces, and maximum attainable rupture speeds. This topic is of particular interest to the exploration of intersonic fault rupture during shallow crustal earthquake events. Working at a different scale to the atomistic studies of Gao et al. (2001), Dunham and colleagues have demonstrated that the transition from subsonic to intersonic propagation can also be triggered by the interaction of a primary crack with a fault plane asperity (Dunham et al., 2003).

Another important topic that raised much interest at the Ringberg Workshop was supersonic fracture, a phenomenon totally unexplained by the classical theories of fracture. Molecular dynamics simulations by the group around Abraham and Gao (Abraham et al., 2002; Buehler et al., 2003) have shown the existence of intersonic mode I and supersonic mode II cracks. This has motivated a recent continuum mechanics analysis of supersonic mode III cracks by Yang (Guo et al., 2003). Recent progress in the theoretical understanding of hyperelasticity in dynamic fracture has shown that supersonic crack propagation can only be understood by introducing a new length scale, called χ , which governs the process of energy transport near a crack tip (Buehler et al., 2003). The crack dynamics is completely dominated by material properties inside a zone surrounding the crack tip with characteristic size equal to χ . When the material inside this characteristic zone is stiffened due to hyperelastic properties, cracks propagate faster than the longitudinal wave speed. The research group of Gao has used this concept to simulate the Broberg problem of crack propagation inside a stiff strip embedded in a soft elastic matrix (Broberg, 1974, 1995). The simulations, which confirmed the existence of an energy characteristic length, are described in Buehler et al. (2003). This study also has implications for dynamic crack propagation in composite materials. If the characteristic size of the composite microstructure is larger than the energy characteristic length, χ , models that homogenize the materials into an effective continuum would be in significant error. The challenge arises of designing experiments and interpretative simulations to verify the energy characteristic length. Confirmation of the concept must be sought in the comparison of experiments on supersonic cracks and the predictions of the simulations and analysis.

While much excitement rightly centres around the relatively new activity related to intersonic cracking, an old but interesting possibility remains to be incorporated in

the modern work: for an interface between elastically dissimilar materials, crack propagation that is subsonic but exceeds the Rayleigh wave speed has been predicted for at least some combinations of the elastic properties of the two materials (Goldstein, 1966).

7. Friction

Friction effects dominate many problems of dynamic fracture, but friction remains one of the least understood aspects of material behaviour. For many material systems, friction involves the contact of more or less intact bodies, whose surfaces are deformed (sometimes severely) by processes such as substrate deformation, asperity contact and wear, wear particle formation, adhesion at newly created surfaces, and plowing of asperities through one another (Anand, 1993; Mora and Place, 1994; Suh and Sin, 1981). Other sliding contact problems, especially in relatively brittle materials, involve layers of rubble or finely comminuted material, which may be pre-existing or created by the sliding process.

Most earthquakes propagate along existing fault lines. The dynamic slip process is complicated by the fact that fault zones contain many rock types, which exhibit distinct mechanisms during crack propagation. The fault zone generally consists of layers of finely granulated rock, created by the fragmentation, melting, and recrystallization of rock during many prior slip events. The fragments or grains within a fault line exhibit a range of sizes extending over one or two orders of magnitude (Chester et al., 1993; Otsuki et al., 2003; Wibberley and Shimamoto, 2003). This suggests that dynamic slip might tend to fragment fault material in such a way that the total fraction of space occupied by the grains is maximized. The grain pattern is reminiscent of the Sierpinski gasket, with smaller grains lying in the interstices of larger grains, in a pattern that recurs down through many length scales. However, the fractal dimension of the grain pattern varies from one slip system to another, suggesting that melting and fragmentation occur at different rates depending on the rock composition in a particular fault. The relation between material composition and the details of nonlinear mechanisms in fault zones, including fragmentation and the resulting distribution of particle sizes, remains obscure.

An analogous situation exists in delamination cracking of many structural materials. Layers of rubble, which have at least superficial morphological similarity, but on much smaller scales, to earthquake fault lines, are also pervasive in predominantly mode II cracks in brittle polymer or ceramic materials, especially delamination cracks propagating between plies in a laminated structure. In such materials, the origins of the rubble can be traced to systems of microcracks that form in layers subjected to shear (Bradley and Cohen, 1985; Fleck, 1991; Xia and Hutchinson, 1994). At large displacements, these microcrack systems coalesce to form entirely detached particles in a layer. Since crack displacements far exceed the particle size in the further crack wake, one infers that shear tractions are transferred across the crack by a mechanism analogous to that acting in the rubble layer in an

earthquake fault line, but with the characteristic length scales being four or more orders of magnitude smaller (Massabò and Cox, 1999; Rice, 1980). Under dynamic loading conditions, adiabatic heating of the rubble layer is, again in attractive analogy to the geological case, likely to cause some melting and therefore rate-dependence through dynamic viscosity.

In both earthquake and structural simulations and models, the action of material along a slip line is represented as friction, i.e., a relationship between the shear traction supported, the prevailing compressive stress across the slip line, and the displacement discontinuity (and its rate, etc.). However, modeling of how the dynamic constitutive behaviour of rubble layers affects friction has not been presented. The constitutive behaviour of granular material is a complicated problem of load transfer through random contacts that change in time. The outcome is strongly affected by the state of hydrostatic compression, rate effects, and the magnitude of the shear displacements. Some work to date has addressed large aggregates of powder (Anand, 1983; Nemat-Nasser, 2000) with applications to ballistics, shear faulting in shock compression, etc. (Nesterenko, 2001). In ballistic applications, constitutive modeling is often reduced to representing comminuted (fragmented) material trapped ahead of a penetrator as a viscous fluid, whose viscosity originates in friction at the contact points of individual particles. In an engineering sense, such models have been at least partially successful in demonstrating the relation between armour design and ballistic performance. However, such constitutive modeling is too crude to permit credible inferences about the role of material type and morphology in determining friction effects in fracture. Extension of the more complete models of granular mechanics to the conditions found in a dynamic crack remains a challenge.

More significant attempts have been made to model friction between deforming (ductile) surfaces, in the absence of comminuted layers, relating friction to the various phenomena of surface deformation (Anand, 1993; Mora and Place, 1994). Such models are now beginning to find application in dynamic fracture (shear instability) studies (Espinosa and Gailly, 2001).

Since the dynamics of friction at the scale of individual grains in a slip line is very complicated, simple Coulomb friction cannot be expected to be correct; both rate effects and nonlinear proportionality will be the norm. Non-Coulombic friction can also be inferred directly from certain characteristics of dynamic mode II crack propagation. Zheng and Rice have shown, e.g., that self-healing or pulse-like crack propagation, which is the prevalent mode of earthquake motion, requires a velocity-weakening friction law (Zheng and Rice, 1998). As reviewed in Section 1, these questions are closely related to whether a frictional sliding problem is ill-posed, which depends on the degree of elastic dissimilarity of the two materials, as well as the nature of the assumed friction law—see the summary table in Cochard and Rice (2000). Understanding how the characteristics of friction that have been deduced from the requirement of well-posedness relate to material processes remains a challenge.

While the simple concepts of fragmentation processes (discussed in Section 2) may provide insight into the first fracture of an elastic material under dynamic overload,

little insight is yet available into the subsequent maturation of first crack systems into granulated material or the progression of large displacements through that material. If earthquake fault lines are taken as indicators, high deformation will create a distribution of grain sizes ranging over several orders of magnitude—there is a tendency away from any single length scale and towards scale invariance. Experiments on granulation effects in polymer and ceramic composites, by which a frictional layer of rubble might be formed, especially in mode II, would be most revealing.

The dominance of friction in various dynamic problems has been well illustrated in recent work. In fibre pullout, e.g., which is the main mechanism of toughening of brittle matrix composites, the presence of friction modifies the nature of the propagating stress wave in two fundamental ways (Cox et al., 2001; Nikitin and Tyurekhodgaev, 1990; Sridhar et al., 2003). Under smoothly varying loading, the furthest propagation of the stress disturbance propagates not at the bar wave speed, but at a reduced speed which depends on the loading rate. Second, when interactions between the loaded fibre and the matrix are taken into account, even simple time-linear loading results in quite complex possibilities of slip, stick, and reverse slip domains, propagating along the fibre–matrix interface. Which pattern of domains is observed depends on the loading rate and material parameters. For loading that varies nonlinearly in time, the patterns of slip become extremely complicated. This character forms a strong contrast with the same problem in the static case, for which elementary solutions, which have been the basis of many engineering material designs, can be written down for arbitrary loading and unloading cycles (Marshall, 1992). Under dynamic conditions, the finite length of a specimen, e.g., in a push-in or pullout test, can also complicate the mechanics of even simple test configurations, through reflected waves (Bi et al., 2002).

One result of particular significance for dynamic fracture in structures relates to laminates in which through-thickness reinforcement is present. Through-thickness reinforcement is often introduced to increase delamination resistance, which it does very effectively. Importantly, experiments and theory have both shown that, for common through-thickness reinforcement types, such as stitches and rods, mode I displacements tend to be suppressed more effectively than mode II displacements, for both static and dynamic loading (Massabò and Cox, 2001; Rugg et al., 2002). Consequently, a strong engineering principle is implied: that robust or damage-tolerant laminated structures will tend to delaminate in mode II and therefore their failure habits will be dominated by friction. This represents a major departure in emphasis from prior work, the great majority of which has studied mode I fracture (where friction is irrelevant). The mode I case is increasingly appearing to be an academic ideal, with the important issues of engineering design depending on mode II behaviour.

8. Problems in materials design and engineering certification

Much of the current engineering relevance of dynamic fracture attaches to structures that must survive either low-velocity impact (especially airframes) or

ballistic assault (military vehicles and ships). Many modern airframes and military structures are fabricated with laminated composites and therefore the principal mechanism of damage that will be driven by dynamic fracture is delamination. Fundamental work on dynamic delamination has been relatively limited, far short of what is needed to understand design principles for structures. Perhaps the discouragement of the complexity of delamination damage has outweighed the attraction of solving such an important problem.

Two features of dynamic delamination damage pose special challenges. First, crack propagation often occurs in nearly mode II conditions; and second, damage almost always proceeds by multiple rather than single cracking. (1) The prevalence of mode II conditions in structures was recognized at least 10 years ago, especially in programs directed by the US Navy, and thence came the intense modern interest in shear crack propagation, intersonic and supersonic propagation conditions, etc., including laboratory and analytical studies and research on earthquakes (see Sections 4 and 5). However, mode II conditions also imply that crack propagation will be dominated by strong friction acting over possibly long intervals of the crack wake. While friction has long been the principal material phenomenon in earthquake modeling, its effect on structural response has received only recent attention. (2) The presence of multiple delamination cracks, which are generally non-symmetrical, being influenced by the orientation of the plies, brings up the difficulty of dealing with very complex crack geometry. Only limited understanding can be expected from experiments or models that invoke plane stress or strain conditions. Much of the response of the structure must be determined by the relationship between the load configuration, the distribution of ply orientations, and the interactions among the numerous cracks that develop.

The recently developed cohesive element computational method offers one promising avenue towards modeling delamination damage in structures. In an early paper, spring elements analogous to the cohesive elements have been used to model the effect of shear tractions (pure mode II conditions assumed) on delamination propagation under static loading (Wisnom and Chang, 2000). This work demonstrated the feasibility of simulating multiple damage mechanisms, especially delamination and splitting cracks, with at least qualitative realism. Elaboration of the solid mesh to allow through-thickness stresses to be computed and of the cohesive traction law to treat mixed mode crack wake processes has also now been demonstrated; the resulting predictions of the evolving shape of multiple delaminations, again under static loading, appear to be very promising (Yang and Cox, 2004). Extension of these methods to dynamic loading is encouraged.

While computationally intense methods provide the only credible simulations of actual structures, analytical methods based on beam theory have proven quite useful in establishing relevant fracture concepts. In the dynamic regime, mode I solutions have been presented for simple delamination problems (Freund, 1993; Hellan, 1978; Kanninen, 1974); and now for cases in which large scale bridging is present due to through-thickness reinforcement (Sridhar et al., 2002). For a wedge loaded specimen, bridging has a profound effect on fracture behaviour, e.g., greatly increasing the crack driving force required to achieve a given crack velocity,

especially at low and moderate velocities. Beam analysis also appears to be remarkably accurate in representing multiple delamination mechanics, at least in the static case. Beam theory predictions of the complicated shielding effects experienced by multiple crack tips in a simple cantilever test case are proven by finite element analysis to be very accurate except when two crack tips are almost coincident in position (Andrews et al., 2004). This is an attractive route to understanding the stability of crack configurations and crack multiplication in an interacting crack population.

For alloys, ceramics, and other non-laminar materials, a significant basis for engineering materials design and qualification already exists from the experimental and theoretical study of materials under shock loading. A comprehensive review may be found in Kanel et al. (1996).

A distinct line of recent research relevant to structures has addressed the fundamentals of dynamic fracture in functionally graded materials. These materials are designed to optimize a given property, such as fracture toughness. Both layered and continuously graded materials have been thoroughly investigated by Shukla and his group (Chalivendra et al., 2002, 2003; Parameswaran and Shukla, 1998, 1999), experimentally and analytically. An important result was that the square-root singularity in the crack-tip fields remains valid, but the higher-order field terms are influenced by the grading of the material. Numerical simulations of the fracture response of such materials under impact loading have been performed recently by Sladek et al. (2004).

Studies of laminated and functionally graded materials are no more than a small gesture to the challenge of simulating dynamic fracture response in structures. Over the last 10 years, by means that have been almost entirely empirical, many ingenious structural designs have been invented and tested for ballistic and impact applications, and also for the containment of explosions, and for enhancing the survivability of civil structures prone to seismic activity or hostile attack. Typically, the inventions address different functions—energy absorption, damage distribution, and remaining strength—by incorporating several material components into a compound system. For example, a ballistic structure may include ceramic tiles, composite layers to encase the tiles, a layer of soft, absorbing material, such as foam, behind the tiles, and a strong stiff backing structure. The empirical approach to optimization, i.e., fabricating and testing such ideas case by case, is very expensive. Much advantage, both in development cost and in the performance that is achieved, would be found in high-fidelity simulations of the entire damage process in such systems, including dynamic fracture as well as other nonlinear, dynamic material responses.

9. Dynamic fracture for manufacture

In recent pioneering work in the field of device manufacture, dynamic fracture has been used to cleave a silicon single crystal along a plane along which a crack cannot normally propagate stably (Current et al., 2001). Cleavage can be assisted by doping

the crystal with impurities using ion bombardment, to change the atomic interactions along the desired plane of fracture. A smooth cleavage can be achieved by loading the crystal dynamically. Non-contact loading can be achieved using air pressure. Such a procedure would appear to be an excellent showcase for the potential of atomistic or quantum simulations for exploring and optimizing a practical process.

An interesting application has also arisen in the cutting of glass by the application of a moving, focused laser beam (Döll and Noll, 2002). A highly localized tension field induced by the focused laser radiation immediately followed by forced cooling drives a brittle crack to propagate, under conditions in which the fracture is a controlled dynamic process stabilized by the local character of the stress field.

10. The future

Research over the last 5–10 years has brought forward numerous fundamental insights into the nature of dynamic fracture. The implications for understanding complex fracture processes at all scales are profound. A particularly rewarding result of the Ringberg Workshop was that, out of many diverse interests, such a strong sense of the universality of dynamic fracture arose that much hope was created for future advances by joint research by people who have not traditionally been associated.

The extraordinary breadth of the physical phenomena and the spatial and temporal scales that are affected by dynamic fracture can perhaps best be stated as a series of questions, drawn from all the different disciplines whose contributions have been reviewed here. The process of answering these questions is bound to lead to major revisions in the way dynamic fracture is perceived, both as a topic in basic science and as a phenomenon that should rightly possess a central place in engineering design principles and reliability codes. A non-unique division of the questions into 10 main categories, with the true complexity of the challenges ahead indicated by the rich subset of problems that lie within each category, is as follows.

1. *When should one use quantum, Newtonian-atomistic, or continuum mechanics?*
What are the limits of validity of continuum mechanics and what determines regimes where either the discreteness of atoms or quantum effects must be accounted for? How does the process of atomic separation at the crack tip couple to quantum excitations—phonons, vibrational modes, electronic transitions, magnetic order—especially in highly confined systems such as nanotubes or nanoscale composites? In quantum simulations, can electron density functional methods do everything? Does core-level relaxation matter during atomic separation? Can continuum methods embellished with cohesive models of displacement discontinuities recreate discrete events in atomistic simulations?
2. *How can computational codes based on continuum, atomistic, and quantum mechanics be tied together into a multi-scale solver?*

How can matching boundary conditions be defined to couple quantum solutions of small groups of atoms to classical atomistic calculations and continuum elasticity?

3. *What is the physics of dynamic fracture in the presence of multiple fields?*
What are the field equations for dynamic fracture when thermal gradients or electromagnetic fields or stress fields from transforming materials are present?
4. *What is dynamic friction?*
What are the fundamental processes subsumed in friction: melting, fragmentation, shear deformation, coupling to normal stresses, coupling to vibrational modes, stick/slip at the atomic scale, or other? What are the constitutive laws for dynamic friction? When are friction problems ill-posed or subject to chaotic behaviour? How do the physical origins and constitutive behaviour of friction control slip nucleation and slip-pulse or crack-like slip behaviour at all scales (atomic clusters to earthquakes?)
5. *How do dynamic cracks propagate in disordered materials?*
What is the role of material heterogeneity, at the atomic scale, or the microstructural scale, or the structural scale, or the geological scale; including crystal defects, dislocations (ductility), voids, and faults? How do the scales of material heterogeneity and the scales implied by wave motion or cohesive separation laws play out?
6. *What controls crack stability and multiple cracking?*
What criteria determine crack arrest, instability with respect to shear, crack branching, crack deflection, and the initiation of detached microcracks? What controls fragmentation in the presence or absence of macroscopic cracks? How do friction or other crack wake mechanisms affect these processes? How do mode ratio effects influence (one suspects very strongly!) crack stability and the mechanics of multiple cracking?
7. *How can new regimes be probed by experiments?*
What novel experiments can exploit different physical phenomena to investigate dynamic fracture? How does one solve the inverse problems posed by the challenge of inferring information about the bulk from boundary observations in the dynamic regime?
8. *What is dynamic crack initiation?*
How can atomic-level descriptions of dynamic crack initiation be reconciled with criteria for initiation used in the continuum regime? What experiments can consistently measure initiation in different materials with different microstructures and degrees of ductility; and be valid in either two- or three-dimensional crack configurations? What engineering standards can be used to characterize dynamic initiation—is dynamic initiation toughness or even the concept of dynamic crack initiation itself useful as an engineering design principle? Is there a material property called crack arrest toughness?
9. *What are the opportunities for exploiting dynamic fracture in manufacture?*
What are the bounds to using dynamic fracture to cleave crystals along desired but non-preferred fracture planes?

10. *How can dynamic fracture modeling be used in designing engineering materials and structures?*

Is it feasible to solve problems of multiple mixed mode dynamic fracture in real structures with sufficient accuracy and speed to optimize the structural response to severe impact or ballistic damage? Can the constitutive laws of friction, microcracking, and other nonlinear effects be inferred sufficiently accurately from tests to enable reliable simulations for design and life certification via dynamic simulations? Can dynamic fracture simulations be integrated with models of fragmentation and the rapid viscous flow of comminuted material? Can standard engineering tests be defined to quantify useful engineering principles such as (if it exists as a material property!) initiation toughness?

The importance of coordinated or collective action for the future must be emphasized. There is a palpable gap between the really interesting new fundamental work that is going on and the needs of engineers in materials design, system certification, and manufacture. While value remains in research that is driven by curiosity alone, the field of dynamic fracture suggests great rewards can also be found by posing basic research problems that respond directly to applications needs.

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