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PLASTIC DILATANCY AS A PARAMETER FOR DUCTILE FRACTURE CRITERION

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Research on ductile tensile fracture processes and available models have shown the important rule of the necked region of tensile specimen, where void growth and coalescence rate is high, due to increased hydrostatic pressure [1-4]. In this case, fracture is associated with a decrease in load bearing capacity of the specimen that coincides with attainment of a critical void volume fraction. Experimental study of tensile fracture of coarse grained cast austenitic (Hadfield) steels has shown that final fracture is not preceded by macroscopic necking. On a microscopic scale, void coalescence does not occur by void sheet formation but rather by internal necking between close voids. Bridging between distant voids becomes possible only after the formation of smaller voids [5]. As a result, the existing models of ductile fracture do not accurately represent fracture phenomena occurring in this class of materials in which necking, loss of load bearing capacity, and void sheet localization phenomena are absent.

Consequently, an alternative approach is proposed, which examines the relationship between plastic dilatancy effects associated with void growth and coalescence, and fracture of the material. The proposal is an "upper bound" approach since localization effects, the role of which is to limit the maximum attainable plastic dilatancy, are not treated.

The new failure criterion proposed, which overcomes the shortcomings of available criteria and models, states that ultimate failure will occur when, for a given increment of plastic strain, the resulting void related plastic dilatancy reaches a critical value which cancels the corresponding increment of plastic work.

An expression for incremental plastic work per unit volume of a plastically dilatant material is given by [6]:

$$dWp = -p\ dV^p/V + \tau d\gamma^p \tag{1}$$

dV^p/V = Plastic dilatancy

p = Hydrostatic component of stress

τ = Shear stress

$d\gamma^p$ = Increment of plastic strain

The plastic dilatancy of a void containing initially incompressible material stems from the growth of existing voids and nucleation of new ones, i.e.:

$$dV^p/V = dV^p/V\ (df,dg) \tag{2}$$

df = increment of void volume fraction due to nucleation
dg = increment of volume due to void growth
Assuming a simple relationship of the form:

$$dV^p/V = df + dg \tag{3}$$

An expression for df has been obtained, assuming strain controlled nucleation and normal distribution around a given nucleation strain [1,2]. In a modified form:

$$df = \left(\frac{1}{E_T} - \frac{1}{E}\right) \frac{f_N}{S_N\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_M^p - \varepsilon_N}{S_N}\right)^2\right] d\sigma_M^p \tag{4}$$

E_T, E	Tangent and Young's modulus, respectively
f_N	Fraction of void nucleating particles
S_N	Standard deviation
ε_M^p	Matrix plastic strain
ε_N	Mean nucleation strain
$d\sigma_M^p$	Increment of matrix plastic strain

An expression for g can be obtained by assuming that void growth is proportional to the matrix longitudinal strain only, i.e., an initial round void transforms into an oval without distortion of the minor axes.

$$dg = \alpha\, d\, \varepsilon_M^p \tag{5}$$

Provided the following matrix plastic stress strain relationship:

$$d\,\varepsilon_M^p = \left(\frac{1}{E_T} - \frac{1}{E}\right) d\,\sigma_M^p \tag{6}$$

And the following simplifying assumptions:

$$p = \sigma/3\ ;\ \tau = \sigma/2\ ;\ \gamma = \varepsilon_o^p/2 \tag{7}$$

where ε_o^p is the external plastic strain.
And replacing (4-7) into (3), one obtains:

$$dV^p/V = f(\varepsilon_M^p)\, d\, \varepsilon_M^p \tag{8}$$

where nucleation and growth terms are grouped into $f(\varepsilon_M^p)$.

Final fracture (for non-zero stress) is defined as the increment of plastic strain for which the corresponding increment of plastic work tends to zero. In other words the available energy becomes kinetic when two broken parts are pulled apart. This definition is proposed as an alternative to the definition employed in numerical calculations where elastic unloading is equivalent to failure. Therefore, until fracture:

$$d\,W_p > 0 \iff d\,\varepsilon_o^p > f(\varepsilon_M^p)\,d\,\varepsilon_M^p \tag{9}$$

Next a simple relationship is assumed between the external and local increments of plastic strain:

$$\frac{d\,\varepsilon_o^p}{d\,\varepsilon_M^p} = T(\varepsilon_o^p) \tag{10}$$

So that the condition for fracture becomes

$$T(\varepsilon_o^p) = f(\varepsilon_M^p) \tag{11}$$

Actual values can be substituted for S_N, f_N, and ε_N [3,4]. Precise determination of α is difficult, but the oval shape of severely deformed voids suggests that α is greater than unity, so that the following approximation is obtained: $f(\varepsilon_o^p) \approx \alpha$ (12)
Consequently the fracture criterion becomes:

$$T(\varepsilon_o^p) \approx \alpha \tag{13}$$

The functional dependence of T on the external plastic strain (although being difficult to determine experimentally) is assumed to be parabolic. For very low strains, T is equal to one, i.e., external and local matrix strains are identical. At higher strains, the external strain includes an increasingly growing contribution of the voids strains corresponding to decreasing contribution from matrix strain.

This model is of course simplified in some aspects, but it has nevertheless the advantage of relating plastic dilatancy to ductile fracture, without additional assumptions about load bearing capacity or void volume fraction. Its numerical verification is expected to yield interesting insights regarding critical values of plastic dilatancy, which are otherwise complicated to determine experimentally. It is meant that density measurements, for example, do not reflect strong local variations (where the failure criterion is fulfilled), as the overall specimen contribution is measured.

In summary, a critical level of plastic dilatancy is assumed to be attained at which the incremental plastic work is reduced to zero. This critical dilatancy which is associated with voids (nucleation, growth) is a general parameter which can be used as a failure criterion.

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