

Bridging thin and thick skin solutions for alternating currents in cracked conductors

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The alternating current potential drop (ACPD) technique is used to monitor surface cracks in electrical conductors. At high frequencies, the current flows in a superficial skin layer. Two distinct solutions are currently available for the thin and thick skin cases. However, there is no general solution that bridges these two modes in a seamless fashion. A numerical model is used to analyze the ACPD of a surface crack with uniform depth. A general solution is given that bridges the thin and thick skin solutions, which closely matches the numerical results, irrespective of the skin thickness. © 2005 American Institute of Physics. [DOI: 10.1063/1.2033131]

Alternating current potential drop (ACPD) is a nondestructive technique used for mapping the depth of a crack by measuring the surface voltage adjacent to this crack.¹⁻⁴ When an AC is passed through a good conductor, the displacement current may be neglected ($\sigma \gg \omega\epsilon$) and the distribution of the electric field in the part is obtained according to Helmholtz Equation:⁵⁻⁷

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t}, \quad (1)$$

where E is the electric field, μ is the magnetic permeability, and σ is the electric conductivity.

When applying high frequency current into metal, the current tends to concentrate in a thin layer close to the surface and flows along the metal surface (skin effect). The skin thickness, δ , depends on the metal properties and the current frequency, f , and is approximated by

$$\delta = \frac{1}{(\pi\mu_r\mu_0\sigma f)^{1/2}}, \quad (2)$$

where μ_r is the relative magnetic permeability and μ_0 is the magnetic permeability of free space.

Figure 1 shows the uniform flow of current in a skin of thickness δ normally incident on a crack of a uniform depth d and width w . The current is injected at point 1 and flows along the metal surface, down and up the crack, and out through point 2. The voltage drop is measured by a probe whose contacts form a gap of constant length Δ . When placing the probe far from the crack, a voltage V_1 is measured. When placing the probe across the crack, a voltage V_2 is measured.¹

The problem simplifies to a *thin skin* problem when the following conditions are met:

$$\frac{t}{\delta} > 10, \quad \frac{d}{\delta} > 10, \quad \frac{w}{d} > 10, \quad (3)$$

where t is the part thickness, d is the crack depth, and w is the crack length.

In thin skin, the electrical field is assumed to be uniform in the region of interest around the crack. This means that the

wavelength along the surface $\lambda \gg d, w$, and Δ . The assumption is probably reasonable for a broad crack ($w \approx d$), but for a very narrow crack ($w \ll d$) the field inside the crack might be partially shielded.

For the thin skin, the crack depth is approximated by³

$$d = \frac{\Delta}{2} \left(\frac{V_2}{V_1} - 1 \right). \quad (4)$$

Equation (4) is referred to as the *one-dimensional (1D) approximation* of the crack depth and gives an excellent estimate for the thin skin case.

By contrast, the thick skin case is met when at least one of the requirements for the thin skin fails to be fulfilled.

For a thick skin, an approximation for the crack depth is given by⁸⁻¹⁰

$$d = \frac{\Delta}{2} \left(\frac{V_1}{V_2} \right)^{1/2} \left(\frac{V_2}{V_1} - 1 \right). \quad (5)$$

Equation (5) derived from calculating the surface electric potential. The potential is of opposite sign on each side of the crack, so that there is a jump in the potential across the crack.⁸⁻¹⁰

In practice Eq. (5) provides a lower limit of the crack depth.

So far, the thin skin approximation is well established and supported experimentally. The thick skin solution is a modification (adjustment) of the thin skin solution that requires cumbersome experimental calibration. But the main problem remains that there is no *seamless transition between these two approximations* that would allow a systematic variation of the experimental parameters. Consequently, ini-

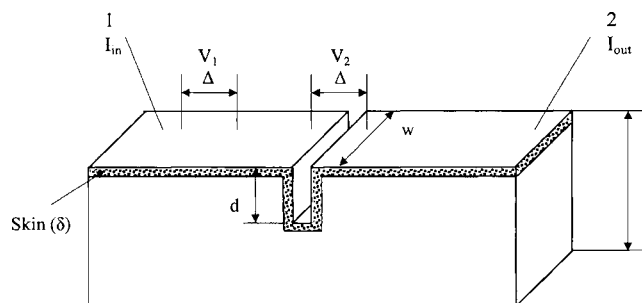


FIG. 1. A schematic description of the ACPD technique.

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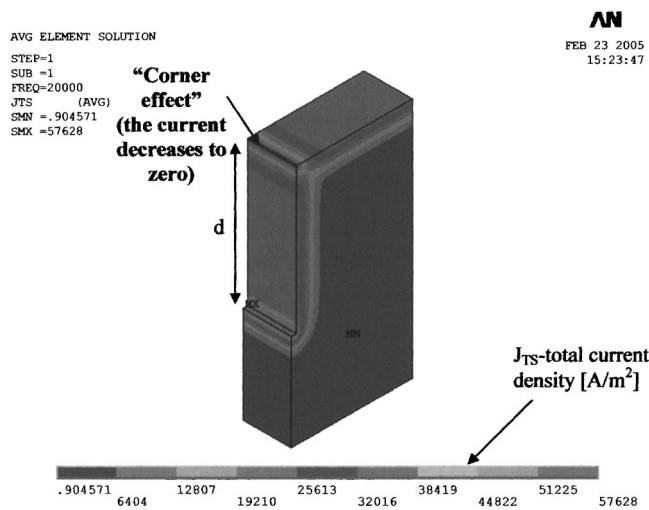


FIG. 2. Current density distribution in a cracked specimen for the thin skin case.

tial “guesswork” is required to determine which skin thickness is most appropriate to the problem at hand.

In this work, we propose a *global solution* which can predict the crack depth for all cases from thin to thick skin in a seamless manner. The solution should provide a good estimate for all probe distances, Δ , especially at short distances, because the sensitivity for crack depth increases as the probe distance decreases.

Maxwell’s equations were solved numerically for various combinations of skin and crack depths using a finite element model (ANSYS).¹¹ The analyzed problem consisted of a metallic conductor with a rectangular cross section area containing a surface crack of uniform depth. A uniform sinusoidal current was applied remotely from the crack, parallel to the surface. The current frequency was varied to model all the cases from thin to thick skin.

Figure 2 shows the current density distribution in a stainless steel plate. The current follows the surface geometry. Near the corner of the crack beginning at a distance of about 2δ from the crack, the current decreases to zero as indicated by an arrow in Fig. 2. A detailed characterization of the current as a function of the distance from the crack (not shown here) shows that the functional relationship is exponential. This phenomenon, subsequently referred to as the “corner effect,” occurs in all the analyzed cases, irrespective of the skin thickness. For the thin skin, the corner effect is very small because its length (2δ) is short compared to the crack depth. The corner effect may therefore be neglected for all practical purposes in this thin skin configuration. At the internal bottom part of the crack, the current increases significantly. This effect stems from the geometrically sharp corners of the present model and is not observed for blunt cracks. This local effect has no real influence on the measured voltage characteristics that are discussed in the sequel.

At distances greater than 2δ , the current density becomes constant, so that the voltage becomes linearly dependent on the distance. The 1D approximation thus gives excellent results when all the thin skin conditions are met.

For the thick skin case, the same phenomenon occurs, but the corner effect is no longer negligible because its characteristic length (2δ) is now of the order of the crack length. Based on these results, we seek a solution based on the 1D solution [Eq. (4)] which also *takes into account the corner*

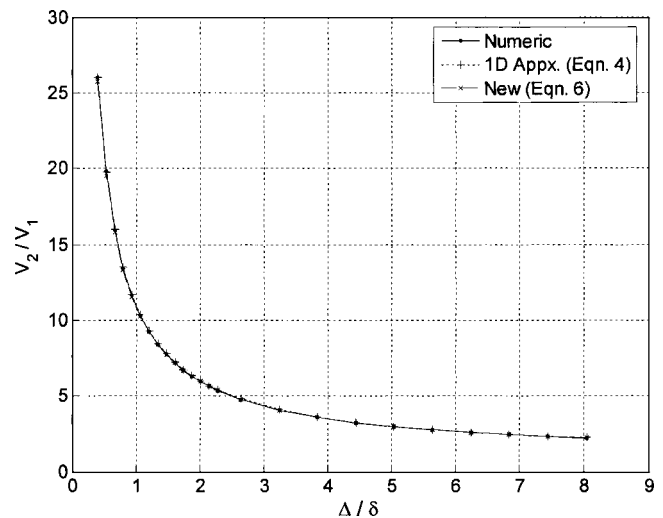


FIG. 3. Thin skin: $t/\delta=50/3$, $d/\delta=30/3$.

effect. Subtraction of the 1D voltage solution from the numerically determined solution yields an exponential relationship between the measured voltages ratio and the normalized probe distance. Therefore, it can be assumed that the corner effect behaves exponentially. By analogy with Eq. (4), the sought solution should have the following form:

$$\frac{V_2}{V_1} = \frac{\Delta + 2d - f_1(\Delta, \delta) - f_2(d, \delta)}{\Delta}, \tag{6}$$

where the functions f_1 and f_2 represent the corner effect. Therefore, f_1 and f_2 are dominant for the thick skin and negligible for the thin skin. The function f_1 compensates for the decrease of the voltage on the specimen surface and is dependent on Δ and δ . The function f_2 compensates for the decrease of the voltage on the crack plane and is therefore dependent on both d and δ .

In all the cases, these functions should have a significant effect at a distance of up to 2δ from the crack, so that the exponential term should also include δ in the denominator.

The proposed empirical equations which include the corner effect is thus given by

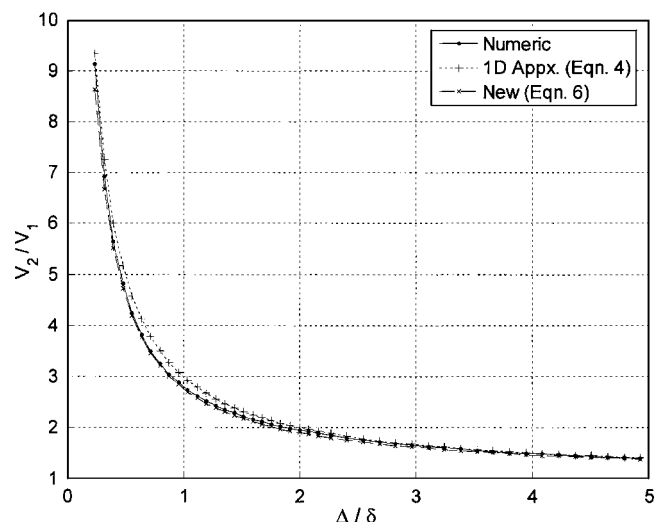


FIG. 4. Thick skin: $t/\delta=50/5$, $d/\delta=10/5$.

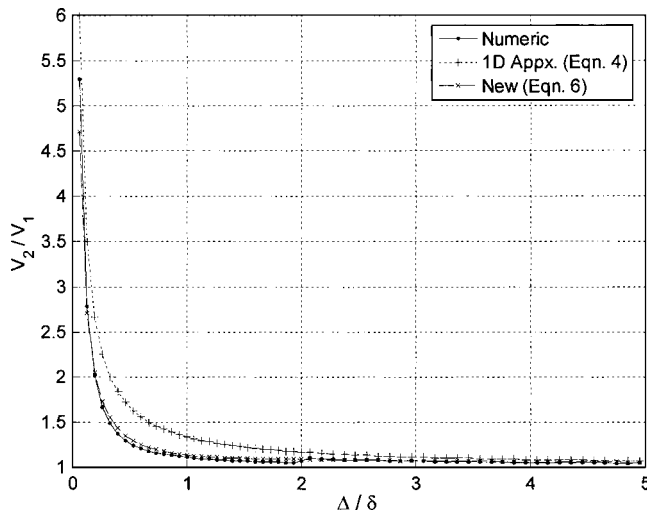


FIG. 5. Thick skin: $t/\delta=50/30$, $d/\delta=10/30$.

$$f_1(\Delta, \delta) = \Delta \cdot \exp\left(-\frac{\frac{\Delta}{2} + \delta}{\delta}\right), \quad (7)$$

$$f_2(d, \delta) = 2 \cdot d \cdot \exp\left(-\frac{d + \delta}{\delta}\right). \quad (8)$$

Simulations were done on different materials (aluminum $\sigma=3 \times 10^7$ s/m and stainless steel $\sigma=1.4 \times 10^6$ s/m), for different crack depth ($0.2t, 0.6t$), and different frequencies (70 Hz–200 kHz) which define the skin depth ($0.1t-t$), and the same of corner effect phenomenon was observed.

Comparisons between the simulation results, 1D approximation, and the proposed solution are shown in Figs. 3–5 for different frequencies and different crack size. These figures show typical results for a thin and thick skin case.

Figure 3 represents a thin skin case and a very good fitting can be noted, as expected, between the 1D approximation [Eq. (4)], the numerical results and the new approximation [Eq. (6)].

In Figs. 4 and 5 it can be observed that, for the same crack depth, the numerical results and the 1D approximation are increasingly divergent as the frequency decreases (thick skin case). The new solution [Eq. (6)] gives a very good fitting for all these cases.

From these results we conclude that the proposed Eq. (6) gives an excellent prediction of the voltage variation in a specimen with a uniform depth crack for all frequencies. This equation is based on the 1D approximation for thin skin, with the addition of the corner effect at a distance of 2δ from the crack. The corner effect occurs at all frequencies, but it is dominant for thick skin and may be neglected for thin skin. Equation (6) is of a high practical interest as it eliminates the need for *a priori* assessment of the skin condition with the appropriate thin or thick skin solution.

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