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A NOTE ON THE DIRECT DETERMINATION OF THE CONFINING PRESSURE OF CYLINDRICAL SPECIMENS

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ABSTRACT

This note presents a simple approach to the direct determination of the confining pressure, q, for a cylindrical specimen encased in a metallic sleeve. The stress analysis of the problem shows that, for a pressure-insensitive material (e.g. metal), q is the quantity by which the stress level of the confined specimen is elevated with respect to the unconfined. As such, q is directly determined by comparing the results of two tests, one with and the second without confinement. For a pressure-sensitive material, q must be determined independently from a plastic stress analysis of the confining sleeve. Then, the same approach can be applied to determine the pressure sensitivity of the material. The present results greatly simplify testing of confined cylindrical specimens for both pressure-insensitive and sensitive materials.

Keywords: confinement, direct determination, metals, pressure, sensitivity

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INTRODUCTION

Many materials are known to be pressure sensitive, meaning that their mechanical behavior depends on the hydrostatic pressure. Testing of these materials over a wide range of pressures requires the application and control of a confining pressure, using several experimental techniques such as hydraulic confinement and hard anvils [see e.g., (Bridgman, 1945)]. Another commonly used method consists of confining a cylindrical specimen by means of a metallic jacket. The latter is usually instrumented with a strain gauge on its external surface, to measure the hoop-strain of the jacket, thereby determining the whole stress and strain tensors of the specimen, and the confining pressure - the reader will find a full account of the stress analysis in Ma and Ravi-Chandar (Ma and Ravi-Chandar, 2000). No restriction is imposed on the state of the jacket, including its possible yielding, although the latter is not recommended by some authors (Ma and Ravi-Chandar, 2000). The technique is well developed and has been applied to the testing of metals and polymers (Ma and Ravi-Chandar, 2000), ceramics (Chen and Ravichandran, 1997; 2000), metallic glasses (Lu and Ravichandran, 2003) and again polymers (Bardia and Narasimhan, 2006).

In a recent investigation of pressure effects on adiabatic shear failure, Hanina et al. (Hanina, et al., 2007) adopted a somewhat different approach in which the confining sleeve is specifically designed to yield at low specimen straining, thus applying a constant confinement provided it is made of a low/non hardening material. While the sleeve can only be used once, the fact that the applied confining pressure is almost immediately constant can be viewed as an advantage over a continuously varying pressure applied by an elastic sleeve.

But the important point is that stress superposition can be taken advantage for the *direct determination of the confining pressure* without the need for attaching a strain gauge to the sleeve. Hanina et al.(Hanina, et al., 2007) exposed the concept very concisely, and the purpose of this note is to develop the basic equations, emphasizing the case of both pressure insensitive and pressure sensitive materials.

STRESS ANALYSIS OF THE SPECIMEN

A cylindrical specimen encased in a metallic sleeve will experience a biaxial confinement *q*, and an axial stress noted σ_2. the stress tensor $\underline{T}$ writes:

$$\underline{T} = \begin{bmatrix} \sigma_2 & & \\ & q & \\ & & q \end{bmatrix} \qquad (1)$$

$\underline{T}$ can be decomposed into its hydrostatic $\underline{H}$ and deviatoric $\underline{D}$ components:

$$\underline{H} = \begin{bmatrix} \frac{1}{3}(\sigma_2 + 2q) & & \\ & \frac{1}{3}(\sigma_2 + 2q) & \\ & & \frac{1}{3}(\sigma_2 + 2q) \end{bmatrix} \text{ and } \underline{D} = \begin{bmatrix} \frac{2}{3}(\sigma_2 - q) & & \\ & -\frac{1}{3}(\sigma_2 - q) & \\ & & -\frac{1}{3}(\sigma_2 - q) \end{bmatrix} \qquad (2)$$

These results are classically used in testing of confined cylinders such as (Chen and Ravichandran, 1997).

Alternatively, we may write:

$$\underline{T} = \begin{bmatrix} \sigma_2 & & \\ & q & \\ & & q \end{bmatrix} = \begin{bmatrix} \sigma_2 - q & & \\ & 0 & \\ & & 0 \end{bmatrix} + \begin{bmatrix} q & & \\ & q & \\ & & q \end{bmatrix} \qquad (3),$$

which is equivalent to writing: $\underline{T} = \underline{T}_1 + \underline{H}_1$.

Note that $\underline{H}_1$ is a hydrostatic tensor. By contrast, $\underline{T}_1$ is not deviatoric, and it can be further reduced:

$$T_1 = \begin{bmatrix} \sigma_2 - q & & \\ & 0 & \\ & & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3}(\sigma_2 - q) & & \\ & -\frac{1}{3}(\sigma_2 - q) & \\ & & -\frac{1}{3}(\sigma_2 - q) \end{bmatrix} + \begin{bmatrix} \frac{1}{3}(\sigma_2 - q) & & \\ & \frac{1}{3}(\sigma_2 - q) & \\ & & \frac{1}{3}(\sigma_2 - q) \end{bmatrix} \qquad (4)$$

which is equivalent to writing: $\underline{T}_1 = \underline{T}_2 + \underline{H}_2$.

Comparing eqns. (3) and (4) with eqn. (2) brings to an identical result, despite the alternative decomposition scheme used here.

But the interesting point here is that inspection of eqn (3) shows that, if the unconfined axial stress is noted by $\sigma_1 = \sigma_2 - q$, then the confined axial stress σ_2 is simply the unconfined stress σ_1 shifted by the confining stress *q*. In other words, this simple exercise shows that *one can directly measure the confining stress q, by comparing two tests, one without and the second with confinement*. This simple fact was used in the study of adiabatic shear banding in magnesium and titanium alloys (Hanina, et al., 2007), but the formal demonstration is presented here for the first time.

EXPERIMENTAL RESULTS

Figure 1 shows typical results of quasi-static testing of AM-50 cylindrical specimens that were encased in 4340 steel sleeves of various thicknesses (Hanina, et al., 2007). Figure 1(a) shows the actual measurement before subtraction of the confining pressure *q*, determined as mentioned above. Figure 1(b) just shows the result of subtracting *q* on the stress level, namely the 3 curves coincide quite well, from the early stages of plastic deformation. Ideally, the 3 curves are expected to be strictly identical, but this could only happen if the confining pressure *q* were constant from the onset of the process. This is not the case, and on the average, for this experiment the yielding process of the sleeve was numerically found to be completed at $\varepsilon \approx 0.05$. Consequently, the negative values of the elastic stresses after subtraction of too large a *q* are not to be considered as physical. One can also note that for this incompressible aluminum-magnesium alloy, the flow curves are essentially similar until the peak stress for all levels of confinement. The effect of the confinement becomes tangible in the strain softening (failure) phase.

DISCUSSION and CONCLUSION

The above mentioned result has a wide practical interest as it allows for a simple and straightforward determination of the confining stress *q* which is the main requirement for the determination of the pressure sensitivity of a material. A constant confining pressure can be applied to a specimen, provided the sleeve's thickness is designed such as to insure early and rapid completion of the yielding process. Another important point to consider in the sleeve's design process is that, for a given sleeve material, the sleeve's thickness will determine the level of applied confinement.

However, the result presented here can *only be applied to pressure-insensitive materials*, e.g. metals.

Consider a pressure sensitive material that can be described by a Drucker-Prager relationship:

$$\sigma(q) = \sigma_0 + \beta q \qquad (5),$$

one can now identify $\sigma_1 = \sigma_0 + \beta q$ to which we add *q* from the $\underline{H}_1$ tensor as before, so that:

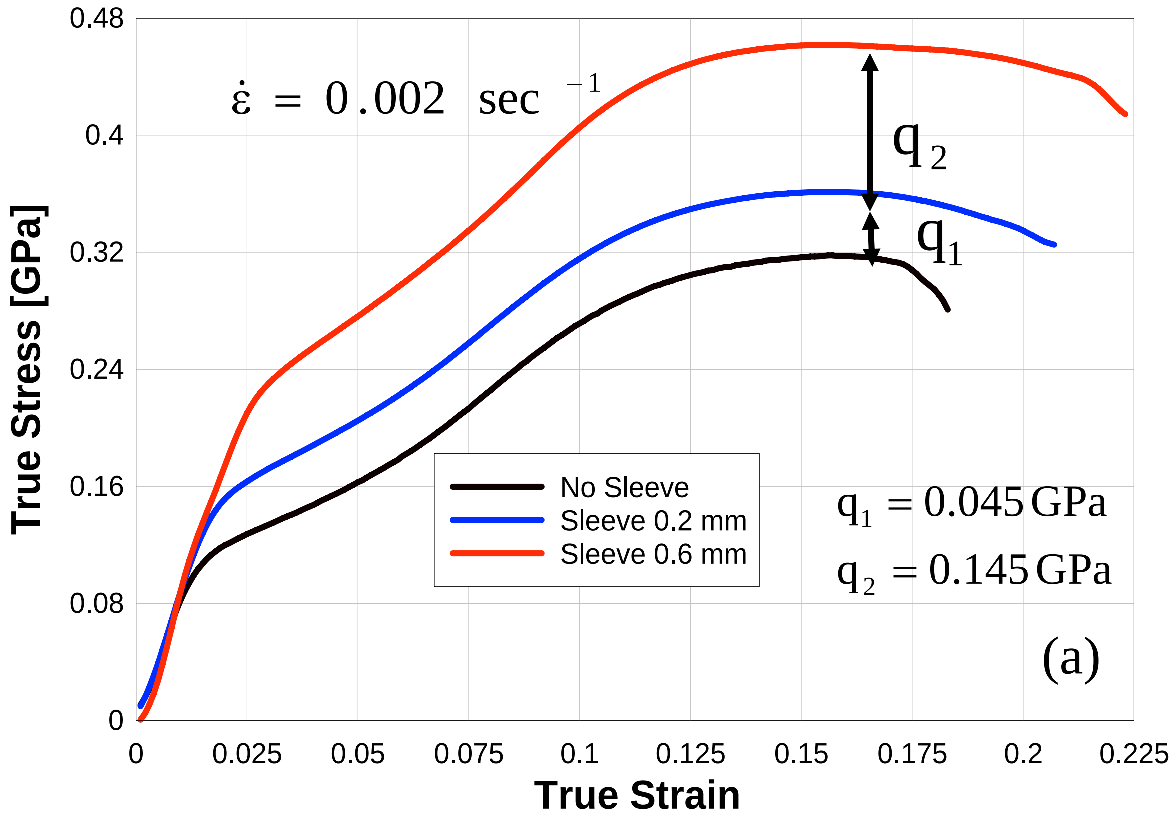
$$\sigma_2 = \sigma_0 + (\beta + 1)q \qquad (6).$$

Eqns. (1) remain unchanged but:

a. Whereas for the Mises material, *q* was directly determined by subtracting σ_1 from σ_2,

b. For the Drucker-Prager material, *q* must be determined (e.g. calculated) separately, as the yielding process of an elastic-plastic pressure vessel (Kachanov, 1974), so that β and σ_0 can now be determined from the $\sigma(q) = \sigma_2 - q = \sigma_0 + \beta q$ vs. *q* (or $p = \frac{1}{3}(\sigma_2 + 2q)$) relationship.

The conclusion of this note is that the testing of confined cylindrical specimens can be significantly simplified based on the present result showing that, for a pressure insensitive material, the confining pressure is determined directly by comparing a confined and an unconfined tests. The approach is also attractive for pressure-sensitive materials, provided the confining pressure is determined independently.



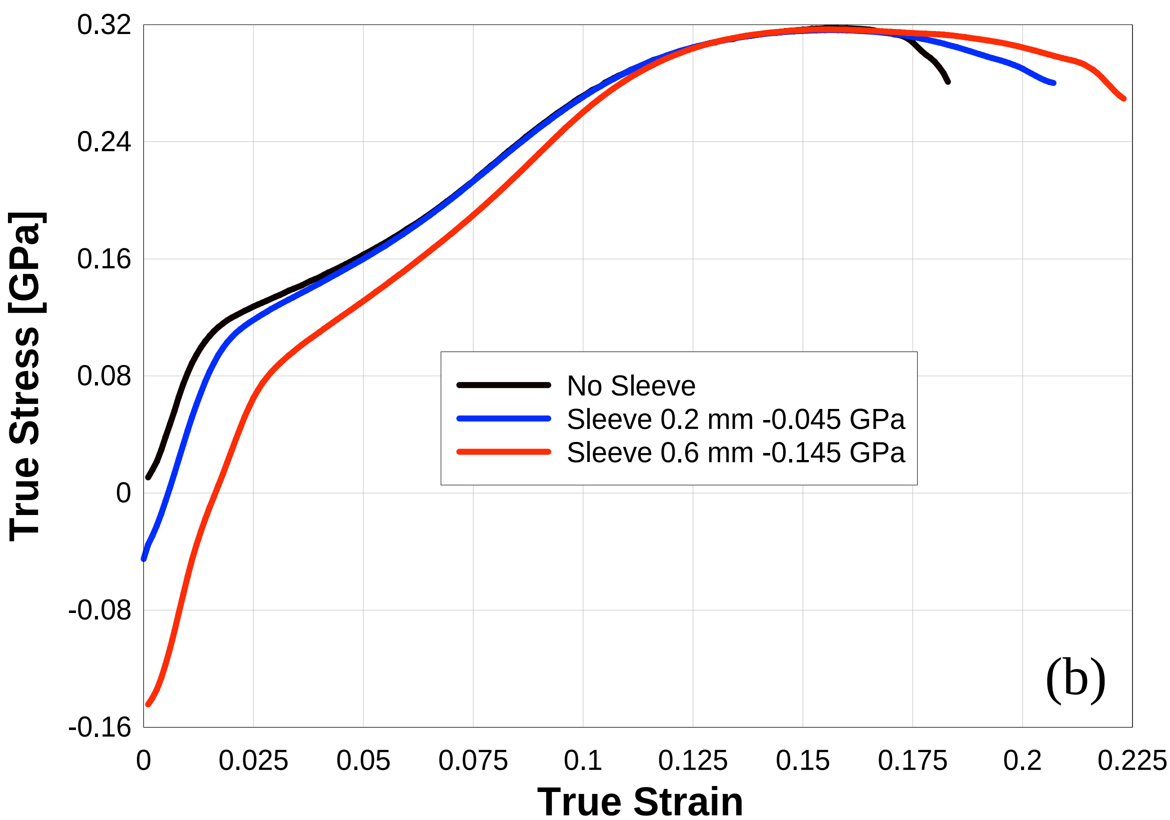


Figure 1: (a) Stress strain curve for quasi-static testing of confined AM50 specimens. (b): like (a) after subtraction of the confining pressure q. Note the similarity of the curves from a strain $\varepsilon \approx 0.05$, beyond which the sleeve is fully plastic, thus applying a constant

confinement until the peak stress. In the elastic regime of the specimen, the confining pressure is not yet constant, and negative stress values in this region are not physical. The post-peak failure stage is found to be pressure-sensitive.

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