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An investigation of the heat generated during cyclic loading of two glassy polymers. Part I: Experimental

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Abstract

A comparative study of hysteretic heating was carried out in commercial polymethylmethacrylate (PMMA) and polycarbonate (PC) specimens subjected to cyclic compressive loading. Both materials heat up upon cycling with a pronounced influence of the cycling frequency and the stress amplitude. Commercial PMMA is very sensitive to minor variations in the maximum applied stress, which does not exceed 0.45 times the yield strength of this material. The temperature rise is continuous throughout the test. The maximum temperature reached is of the order of the glass transition temperature (T_g), and often more. Failure is sudden and consists of localized bulging in the central part of the specimen. Commercial PC can be tested at much higher stress levels, of the order of its yield strength. Here, a well-defined temperature peak, which has not been reported previously, develops during the initial stage of the loading. The maximum temperature reached during the test does not exceed $0.8T_g$. The sharpness of the peak improves with increasing stress amplitude and testing frequency. Failure of the specimen occurs by diffuse barreling. Annealing heat treatments shorten the fatigue life of PMMA specimens and decrease the sharpness of the thermal peak of PC specimens. A non-uniform temperature distribution is observed to develop in the specimen during cycling. Consequently, care should be paid to the thermal boundary conditions of the problem. The failure mechanism (diffuse vs. localized) of the investigated materials is thus dictated both by the temperature distribution and by the extent of the temperature rise. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Cyclic loading of a viscoelastic or elasto-plastic material (beyond the elastic domain) yields a hysteresis loop in the stress–strain relationship. Such a loop indicates that part of the strain energy is not recovered but dissipated during the cycle. Taylor and coworkers (Taylor and Quinney, 1934; Farren and Taylor, 1925) showed that a large part of the mechanical energy converts into heat. It was later shown that the fraction of plastic work converted into heat is both strain and strain rate dependent (Mason et al., 1994; Rittel, 1999). When the heat is not allowed to flow out of the structure, as in adiabatic conditions, the temperature of the material increases, sometimes in noticeable proportions (Chou et al., 1973). While this effect is quite remarkable at high rates of loading, it may also be

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significant when energy is constantly input into the structure as in the case of cyclic loading. Numerous experiments on temperature changes during cyclic loading of metals have been carried out (see e.g., Dillon, 1966, 1976). Concerning polymeric materials, the effect of hysteretic heating has been clearly shown to dramatically affect the mechanical response of the material (Riddell et al., 1966; Constable et al., 1970). The latter showed that polymethylmethacrylate (PMMA) specimens could reach the glass transition temperature (T_g) of this material when loaded to a maximum cyclic stress of about 0.35 times its yield strength. When testing was carried out at a lower stress level, the temperature would increase at first and it would then level off. Tauchert and Afzal (1967) reported similar observations for tubular specimens of the same material (see also related work on polyethylene by Tauchert (1967)). Li et al. (1995) investigated the fatigue properties of polycarbonate with emphasis on cyclic damage (volume changes) in cyclic tension. In their experiments, the cyclic stress amplitude did not exceed $0.5\sigma_y$. Lesser (1996) noted that the material behavior in the low or high cycle fatigue regime involves volume changes during the deformation process. He thus distinguished two domains in the fatigue life of the specimen: the thermally dominated region (maximum stress amplitude of 50 MPa ($0.7\sigma_y$)–60 MPa ($0.85\sigma_y$) for his polyacetal) and the mechanically dominated region at lower stresses. The distinction between the domains depends not only on the stress level but also much on the cycling frequency. Finally, the stability of the temperature reached was investigated and discussed recently by Molinari and Germain (1996). In all the above-mentioned sources, the typical response of a cyclically loaded polymer is that the specimen temperature increases rapidly with the cycling, then it does (or does not) stabilize until final failure. *The process is supposed to be smooth as there is no reason to suspect a discontinuous behavior of the temperature rise.*

Most of the previous work concerned tension, alternated tension-compression, or torsion testing. In this case, the applied stress may either be non-uniform and/or limited by the development of a tensile instability. The present work addresses compression testing, for which the stress is *at the same time* uniform (as long as geometrical instabilities are excluded) and it is not limited by the development of a tensile instability, even at relatively high stress levels of the order of the yield strength. We investigate and compare systematically the cyclic compression behavior (mechanical and thermal) of two polymers, commercial PMMA and commercial polycarbonate (PC). Both materials are tested at the higher stress levels which can be sustained without immediate specimen failure. The present study confirms and extends previous results about PMMA. For PC, when the stress amplitude exceeds $0.8\sigma_y$, *a new and previously unreported sharp temperature peak* (*discontinuity*) *develops during the first few thousands of cycles.* Temperature levels off subsequently, until it rises again towards failure.

The present paper is organized as follows: first, the experimental setup, basic theoretical background and data reduction techniques are exposed. Characteristic results are then shown and discussed for the two materials. The evolution of the specimen temperature is characterized with respect to the stress amplitude, frequency, annealing heat treatments, and interrupted loading. The observed failure modes are addressed in relation with the temperature distribution. Finally, conclusions are drawn about the nature of hysteretic heating in the investigated polymers.

In the following paper, we further investigate and model the conversion of mechanical into thermal energy in these materials.

2. Theoretical background

Consider a viscoelastic material with a complex stress relaxation modulus G^*, that is $G^*(\omega) = G_1(\omega) + iG_2(\omega)$, where ω is the angular frequency (Ward, 1971). For a given strain $\varepsilon = \varepsilon_0 \sin \omega t$, the corresponding stress is given by $\sigma = \sigma_0 \sin(\omega t + \delta)$, where δ is the phase angle between the applied strain and the resulting stress. The strain energy is given over one period by

$$W = \int_0^T \sigma \, \mathrm{d}\varepsilon = \omega\varepsilon_0^2 \int_0^T (G_1 \sin\omega t \cos\omega t + G_2 \cos^2\omega t) \, \mathrm{d}t. \tag{1}$$

The first term of the integral is equal to 0, meaning that it does not contribute to the dissipation of energy during the cycle. By contrast, the second term is not equal to 0 and this gives an expression for the dissipated energy

$$W_{\mathrm{in}} = \pi G_2 \varepsilon_0^2. \tag{2}$$

Recalling that $G_2 = (\sigma_0/\varepsilon_0)\sin\delta$, Eq. (2) becomes

$$W_{\mathrm{in}} = \pi\sigma_0\varepsilon_0 \sin\delta. \tag{3}$$

The value of $\sin\delta$ is obtained from the hysteresis loop formed by plotting the stress vs. strain. Consequently, for selected points during the test, the dissipated energy is obtained by evaluating W_{in} through Eq. (3). The effects of the frequency is now assessed by dividing W_{in} by one period, $2\pi/\omega$, to yield $\dot{W}_{\mathrm{in}}$. The resulting power increases with the cycling frequency. This naturally emphasizes the role of the testing frequency. The convertibility of mechanical energy into heat is expressed in the coupled heat equation. For a linear viscoelastic solid, this equation writes

$$s_{ij}^{\mu}\dot{d}_{ij}^{\mu} + k\Delta T = \rho c_E \dot{T} + (3\lambda + 2\mu)\alpha T_0 \dot{\varepsilon}_{kk}, \tag{4}$$

where k is the thermal conductivity of the material, c_E the specific heat at constant deformation, λ and μ are Lamé constants. A superimposed dot indicates derivative with respect to time. The product of the deviatoric strain component s_{ij}^{μ} by the viscous strain rate $\dot{d}_{ij}^{\mu}$ expresses the rate of energy dissipation, when summation has been assumed over the μ branches of the rheological model which describes the solid (Boley and Weiner, 1960). The rightmost term of Eq. (4) expresses the fact that volume changes contribute to the change of thermal energy (thermoelastic, thus reversible, effect). The rate of mechanical energy dissipation is a fraction (β) of the power enclosed in the hysteresis loop ($\dot{W}_{\mathrm{in}}$). The complementary part, $(1-\beta)\dot{W}_{\mathrm{in}}$, is stored in the material through microstructural changes (see the review by Bever et al., 1973). Rittel (1999) has shown that the ratio of the thermal to mechanical energies (and powers) is not constant in polymers, and it varies as a function of the strain and strain rate. However, to a first approximation, this ratio will be assumed to be a constant for small viscoelastic strains and negligible thermoelastic effects. Eq. (4) can now be rewritten as

$$\beta\dot{W}_{\mathrm{in}} + k\Delta T = \rho c_E \dot{T}. \tag{5}$$

Therefore, the temperature distribution in the specimen can be calculated by solving Eq. (5) with the appropriate boundary conditions for heat transfer once $\dot{W}_{\mathrm{in}}$ has been measured. It can be noted that, when adiabatic conditions prevail, Eq. (5) gets simpler as $k\Delta T = 0$. While adiabatic assumptions are justified for transient loading, they do not apply to the present case of sustained cyclic loading.

3. Experimental

The polymers chosen in this study were cylinders of commercial PC and commercial PMMA. All the specimens were machined from the thickness of polymeric plates. Typical specimen dimensions were 12 mm height and 15 mm diameter. Temperature variations throughout the test were monitored by means of a small embedded (T-type) thermocouple. The sensing tip was located at mid-height of the cylinder at a depth roughly equal to the radius. A small hole was first drilled, the thermocouple was then inserted and the cavity was sealed by introducing a droplet of home-made (dissolved chips) matching liquid polymer. The technique has been previously used by Rittel (1998a,b) in his study of transient thermal phenomena related to dynamic fracture. The output voltage from the thermocouple was fed differentially into a Nicollet 490 digital oscilloscope with a sampling resolution of 12 bits.

The specimens were loaded in compression by a prescribed sinusoidal load at various frequencies (3, 10 and 15 Hz) using a servo-hydraulic tensile machine (MTS 810). A slight compressive preload was applied to ensure sustained contact with the specimen. The upper and lower faces of the specimens were lubricated with petroleum jelly (Walley et al., 1989) and 5 mm thick alumina plates insulated the specimen from the steel platens. In several experiments, the specimens were slipped into a polymeric foam jacked to minimize radial heat losses by convection. Both the load and the crosshead displacement were monitored throughout the experiment. It is reasonably assumed that the system is much more rigid than the compressed cylinder. Loads and displacements were later converted into (nominal and true) stress and strain. The experiments consisted of continuous monitoring of the prescribed load, resulting displacement and temperature (at the center of the specimen and sometimes near its end) until final failure or deterioration of the thermocouple.

4. Results

4.1. Static results

Monotonic compression tests and temperature measurements were carried out under displacement (crosshead velocity) control. The corresponding typical (nominal) strain rates were $\dot{\varepsilon} = -0.14$, -0.014 and -0.003 s^{-1} (Fig. 1). The results show that the yield strength of both PC and PMMA increases with the strain rate. A first estimate is that for PC, it increases at a rate of about 10 MPa/strain rate decade, starting at about 60 MPa. For PMMA, the yield strength increases at a rate of about 20 MPa/strain rate decade, starting at about 100 MPa. In the sequel, a *representative* value for the yield strength under cyclic loading will be assumed to be $\sigma_y = 70$ MPa for PC and $\sigma_y = 120$ MPa for PMMA.

The corresponding temperature changes are shown in Fig. 2 for the two materials. The temperature increases with the strain rate as the deformation gets closer to adiabatic. At the lowest strain rate, the temperature does not increase markedly. Our results are in full qualitative accord with those of Arruda et al. (1995). However, the temperature rise measured in the core of our specimens is slightly higher than that reported by these authors. One reason may be that we used embedded thermocouples while they used infrared sensing to determine the surface temperature of the specimen (the issue of surface vs bulk temperature measurements is discussed in Kapoor and Nemat-Nasser (1998)). It is also noted that with increasing strain rate, the slope of the temperature as a function of the strain increases. An additional point is that the threshold strain at which the temperature starts to rise is greater at higher strain rates. For PC, this threshold is of the order of $\varepsilon \approx 0.15$ whereas for PMMA, it exceeds $\varepsilon \approx 0.2$. These observations are in agreement and complementary to previous high strain rates observations by Rittel (1999). In this previous work, the noticeable temperature rise had been associated with the softening part of the stress–strain curve of the material.

4.2. Hysteretic heating at various stress levels and frequencies

First, typical results obtained for PC specimens are shown in Figs. 3–5. The general tendency is that the temperature rises with cycling, stabilizes during a large number of cycles, and then starts rising again toward final failure. This observation goes along with an increase of the average displacement (also called 'dynamic creep'). In the lower part of Fig. 3 (disk 4), the frequency has been increased from 10 to 15 Hz after some 9×10^4 cycles. As a result, both the temperature and the creep rate increase simultaneously. The effect of the stress amplitude is illustrated in Fig. 4. As the maximum stress level exceeds $0.8\sigma_y$ a well-defined temperature peak appears, during approximately the first 10^4 cycles. The maximum recorded temperature exceeds 70°C after a relatively small number of cycles (about 4000). The amplitude and the sharpness of the peak increase with the applied stress level. The influence of the frequency is illustrated in Fig. 5. The sharpness of the initial temperature peak increases with the cycling frequency.

Next, typical results obtained for PMMA specimens are shown in Figs. 6 and 7. As in the case of PC, the temperature increases with the stress amplitude and with the frequency. These results are identical to those of Constable et al. (1970). However, one should remark that a small

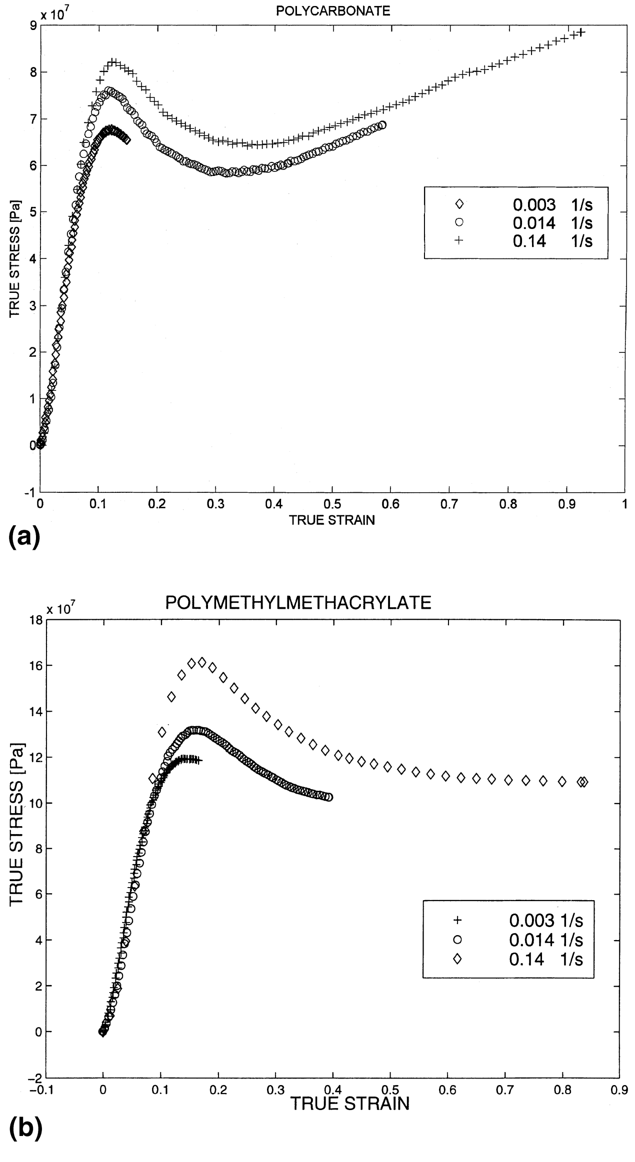

Fig. 1. Monotonic true stress–true strain curves for polymeric disks tested in compression at various strain rates: (a) PC; (b) PMMA.

variation in the nominal cyclic stress, from 0.4 to $0.44\sigma_y$, reduces radically the life of the specimen.

A first important distinction between the two polymers is that the temperature peak which characterized PC is not observed for PMMA. The PC specimens were stressed in the close vicinity and sometimes above its yield strength. A very interesting feature of this peak is that the temperature drops to form a plateau rather than increasing rapidly beyond bounds. Such 'regularization' may indicate that the material cyclically hardens so that less and less mechanical energy is dissipated upon continued cycling. By contrast, the PMMA specimens were tested at much lower relative stress levels of the order of $0.4\sigma_y$ and below. For this material, it was observed that increasing the maximum stress level slightly beyond this limit resulted in immediate collapse of the specimen.

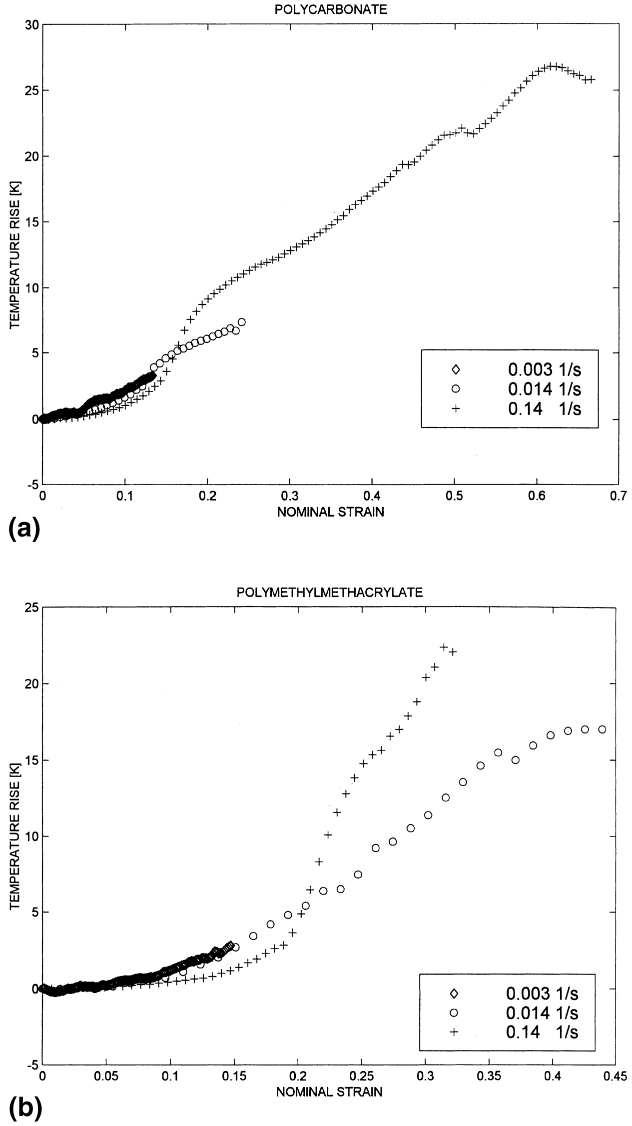

Fig. 2. Evolution of the core temperature as a function of strain for the experiments shown in Fig. 1: (a) PC; (b) PMMA.

One additional important point concerns the homologous temperature (T/T_g) reached as a function of the relative applied stress. For PC, whose glass transition temperature (T_g) is about 423 K, cycling at a relative stress level of the order of σ_y brought to a maximum temperature *rise* of about 50 K, so that the maximum homologous temperature reached $0.8T_g$. For PMMA, whose glass transition temperature is about 373 K, the observed temperature rise of about 50 K and sometimes much larger while the relative stress level was low. Such a temperature rise corresponds to a homologous temperature of $0.92T_g$ and above. In other words, PMMA reaches a much higher homologous temperature than PC for a lower maximum relative stress.

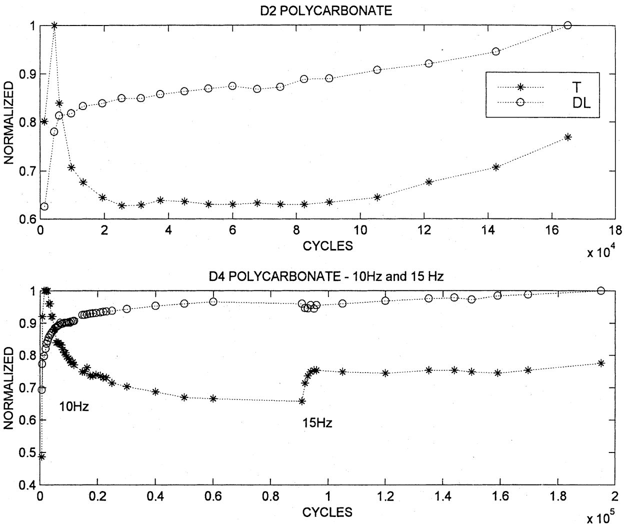

Fig. 3. Cyclic loading of PC disks. Results normalized with respect to their maximum value. Upper frame: Evolution of the temperature and the average shortening as a function of the number of cycles. Note the initial thermal peak. Lower frame: Same experiment where the frequency was changed from 10 to 15 Hz. Note the step temperature rise.

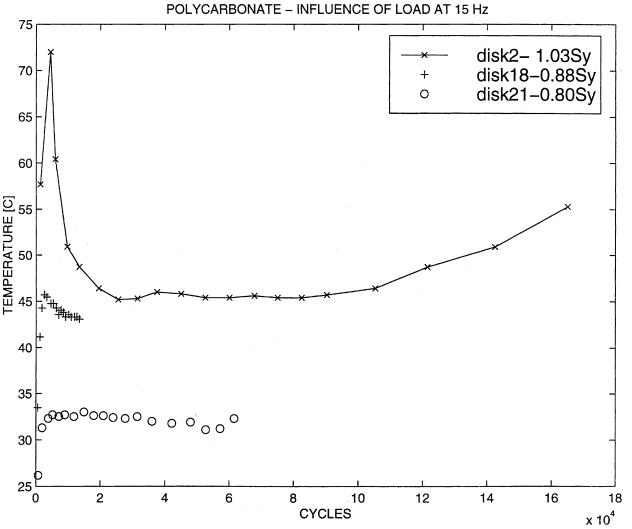

Fig. 4. Cyclic loading of PC disks. Influence of the maximum nominal stress on the temperature rise. The thermal peak is increasingly defined at higher stress levels.

4.3. Size effect and temperature uniformity in the specimen

Specimen final failure is characterized by a sudden collapse of the cylinder. As shown in Fig. 8, the failure mode is rather diffuse for PC and it corresponds to barreling. By contrast, failure of the PMMA specimens is quite localized, and consists of a torus-like bulging at the specimen mid-height. The torus-like region is a bit less

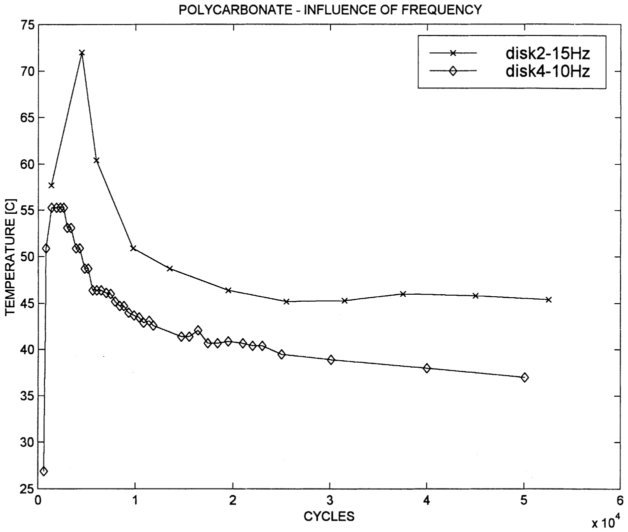

Fig. 5. Cyclic loading of PC disks. Influence of the frequency on the thermal response. The stress level is identical for the two specimens.

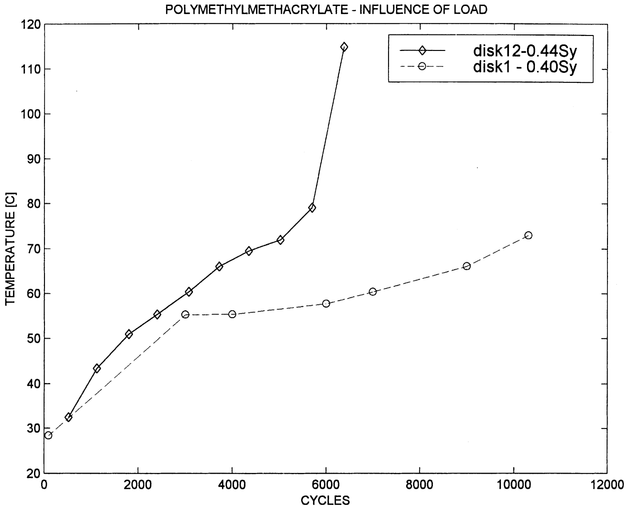

Fig. 6. Cyclic loading of PMMA disks. Influence of the maximum nominal stress on the temperature rise. Note the high sensitivity of the material to minor changes in the stress level.

transparent than the unstressed material. It is enclosed between two (upper and lower) cylindrical regions which remain transparent (Fig. 8). This localized failure mechanism indicates a potential lack of temperature uniformity in the specimen(s).

To verify this point, we embedded one additional thermocouple at about 1 mm depth and 1 mm from the lower side of the cylinder (Fig. 9). By contrast with the core thermocouple, the surface thermocouple is located such as to be influenced by radial and longitudinal heat transfer. Typical results are shown in Figs. 10–12 for PC specimens. First, two specimens with different diameters were stressed at identical stress levels. A comparison of Figs. 10 and 11 shows that regardless of the specimen diameter, the near surface thermocouple

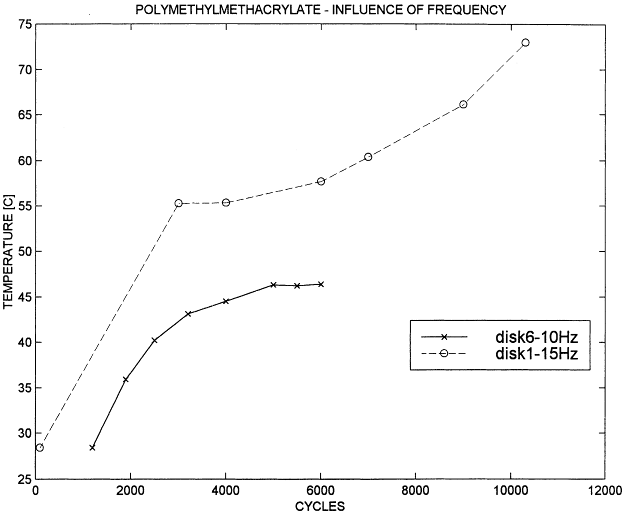

Fig. 7. Cyclic loading of PMMA disks. Influence of the frequency on the thermal response. The stress level is identical for the two specimens.

always senses lower temperatures than the core thermocouple. The peak, which is detected by the core thermocouple, does not appear in the near surface reading. Beyond the initial peak, the temperature difference does not exceed 5 K, which corresponds to a relative difference of the order of 0.15. In Fig. 12, the readings of core thermocouples are plotted for two different specimen diameters with a common testing frequency. As the relative stress is low, the peak is rather ill defined but it nevertheless appears in both specimens, regardless of their diameter.

It can thus be concluded from these experiments that the specimen temperature is indeed inhomogeneous so that the boundary conditions, which cause radial and axial heat conduction, are important. This point will be further addressed in the second part of the paper. The higher temperature is sensed in the core of the specimen, regardless of its diameter. These observations corroborate previous calculations by Tauchert (1967) for cyclic torsion, and by Arruda et al. (1995) for monotonic compression. Consequently, the failure mode, whether diffuse or localized, certainly reflects the inhomogeneous temperature distribution, on the one hand. On the other hand, the extent of the localization is dictated by the homologous temperature reached in the central part of the specimen. A higher homologous temperature was observed to correspond to a localized type of failure mechanism. (A detailed study of the localized failure mode in PMMA will be reported in a subsequent paper.)

4.4. The influence of thermal treatments on hysteretic heating: cyclic and monotonic response cyclic

In order to further characterize the nature of the temperature peak in PC a series of experiments was undertaken which involved annealing heat treatments and reloading of the specimen. The annealing treatment was applied in three different cases.

In Fig. 13, the thermal response of disk 24, which was annealed in the post-peak regime (thermal plateau), is shown. The specimen was held at 120°C (about $0.92T_g$) for 100 min, followed by furnace cooling. Upon reloading, a well-defined peak does not form and the temperature plot shows an ill-defined peak spanning over 10.000 cycles. Beyond this point (cumulative 35.000 cycles), the thermal curve is the continuation of the original curve prior to annealing.

Disk 16 was first loaded until the temperature reached its maximum (Fig. 14 and comparison

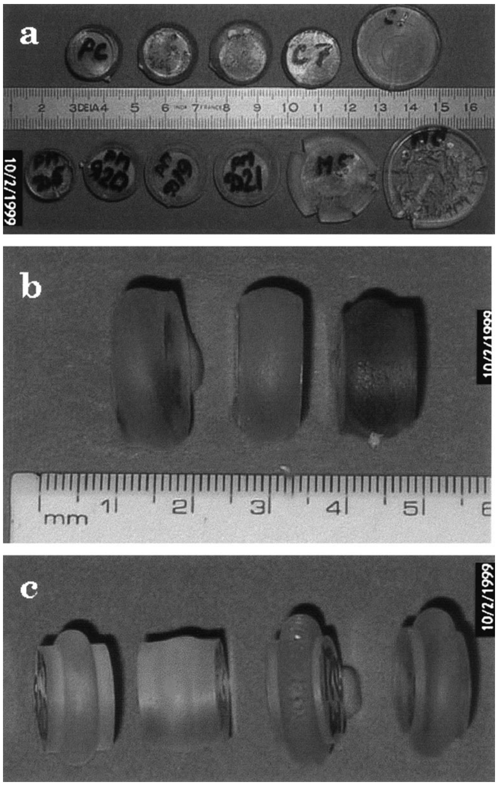

Fig. 8. (a) Upper view of selected failed PC (upper row) and PMMA (lower row) specimens. The right-hand side larger specimens were tested in monotonic compression. (b) Side view of failed PC specimens. Failure occurs by a *diffuse* barreling failure mechanism. (c) Side view of failed PMMA specimens. Note the *localized* bulging failure mode.

with Fig. 4). It was subsequently annealed at 120°C for 100 min, then furnace cooled. The specimen was then reloaded to various successive stress levels. Fig. 14 shows that at first reloading following annealing, the specimen does not display a temperature peak even if the relative stress level has been increased from 0.79 to $0.87\sigma_y$. Rather, the maximal temperature reached is lower by about 5°C (from 40°C to 35°C). The peak reappears during the subsequent loading stages when the relative stress noticeably exceeds the yield strength of the material.

Finally, in Fig. 15, we have plotted the response of two disks (24 prior to annealing and 25). Disk 25 was annealed prior to initial loading following a procedure identical to that of disk 24. While the post-peak behavior of the two specimens is virtually identical, one can clearly note that the annealing treatment reduces the amplitude of the thermal peak by smoothing it, without erasing it totally for this specific treatment.

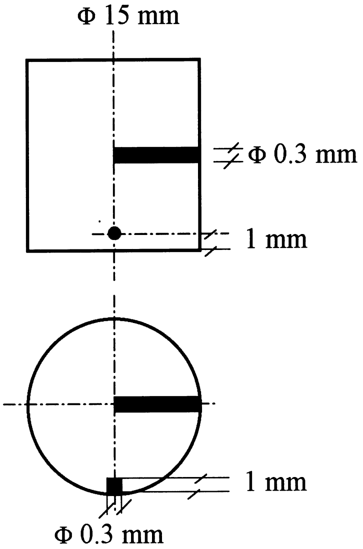

Fig. 9. Schematic representation of the location of the embedded thermocouples used to assess the temperature inhomogeneity in the specimen.

In Fig. 16, results are plotted for PMMA specimens, with and without annealing. The annealed specimen (disk 5) was heated to 110°C (about $1.02T_g$) during 300 min, followed by furnace cooling. One obvious outcome of this heat treatment was that the fatigue life of the specimen was considerably reduced w/r to the un-annealed material. It must be noted that this kind of accelerated failure is observed to occur at higher relative cyclic stresses. There is no evidence that the heat treatment lowered the yield strength of the PMMA. However, keeping in mind the high sensitivity of this material to small variations in the stress level, it appears that a slight decrease in the yield strength would dramatically shorten the life of the specimen.

4.5. Monotonic

The influence of annealing treatments on PC has been investigated by Golden et al. (1967) and

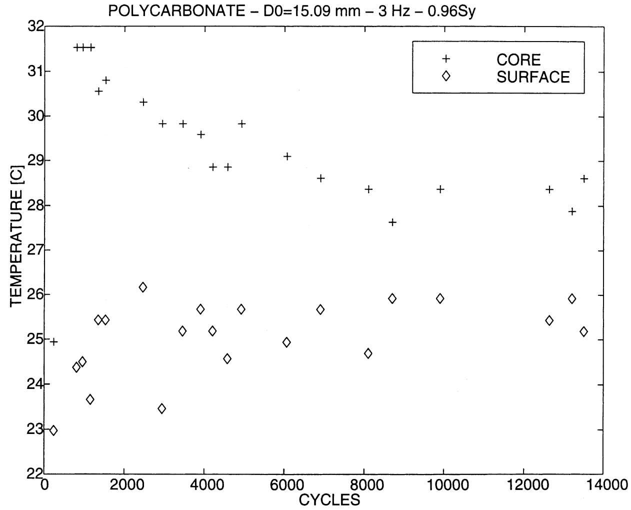

Fig. 10. Cyclic loading of a PC disk. Evolution of the core and near-surface temperatures as a function of the number of cycles.

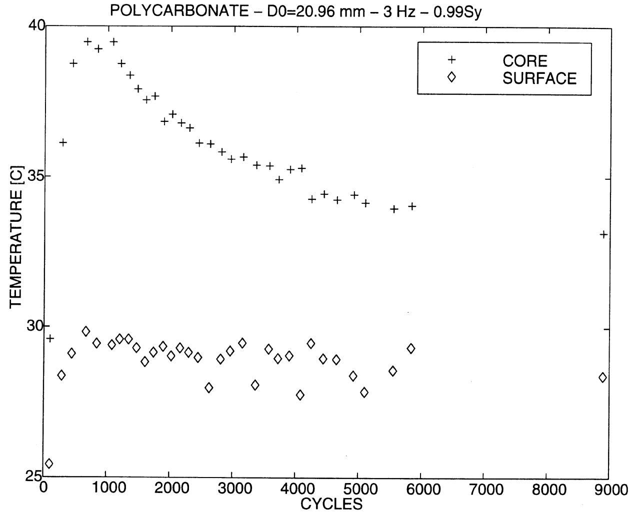

Fig. 11. Cyclic loading of a larger diameter PC disk. Evolution of the core and near-surface temperatures as a function of the number of cycles, as in Fig. 10. The diameter of the disk is greater than that of Fig. 10 but the stress levels are identical.

later by Adam et al. (1975). One of their results was that the annealing treatments increase the yield strength of the material. However, this increase was less noticeable for a commercial grade of PC, when compared to their 'home-made' PC. Consequently, we subjected our commercial PC and PMMA materials to compressive (and thermal) testing to assess the effect of the heat treatment. Figs. 17 and 18 show the results of compressive tests carried out on annealed and not annealed specimens of PC and PMMA. These figures clearly show that the heat treatment does not affect the compressive properties of the material, at least to a macroscopic level. Consequently, one can reasonably assume that the heat treatment did not affect the yield strength of the investigated

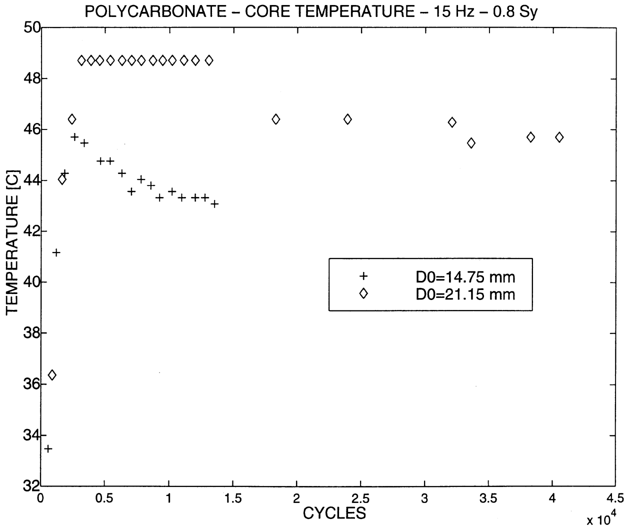

Fig. 12. Cyclic loading of PC disks. Evolution of the core temperatures for a smaller and larger diameter disks.

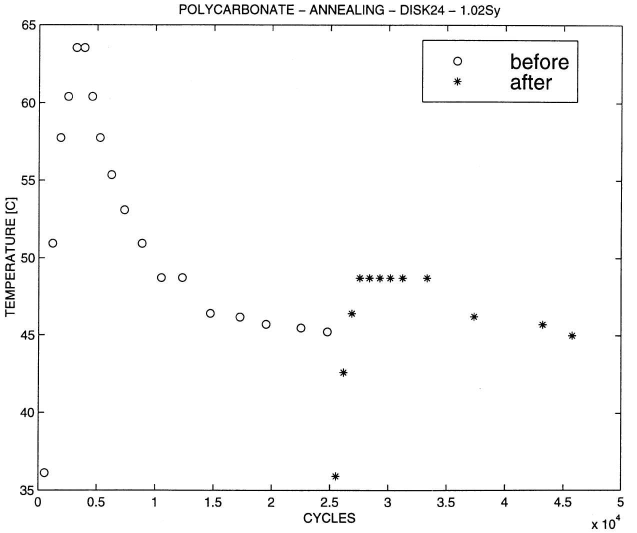

Fig. 13. Cyclic loading of a PC disk. Evolution of the core temperature. The specimen was heat-treated and cyclically reloaded to the previous stress level . Note the ill-defined thermal peak upon reloading. At a later stage, the temperature evolves along its original curve.

materials to a noticeable level. This observation corroborates the previously mentioned observation of Adam et al. (1975). It can be noted that the thermal response of the materials does not seem to be influenced by the thermal treatment, as well.

5. Discussion

The experiments which have been reported in this paper concern the hysteretic heating of two commercial polymers, PMMA and PC. Cyclic

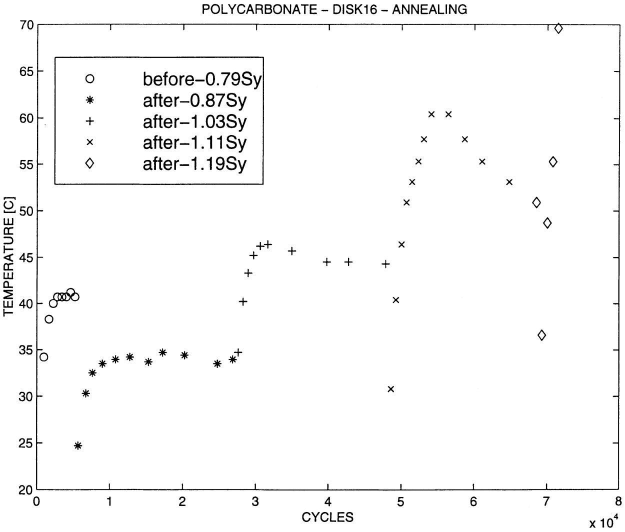

Fig. 14. Cyclic loading of a PC disk. Evolution of the core temperature. The specimen was first loaded until a maximum temperature was reached. It was then heat-treated and cyclically reloaded to successive increasing stress levels. The specimen responds as if it were tested to an overall lower stress level. The peak appears anew when the stress level is quite high $(1.11\,\sigma_y)$.

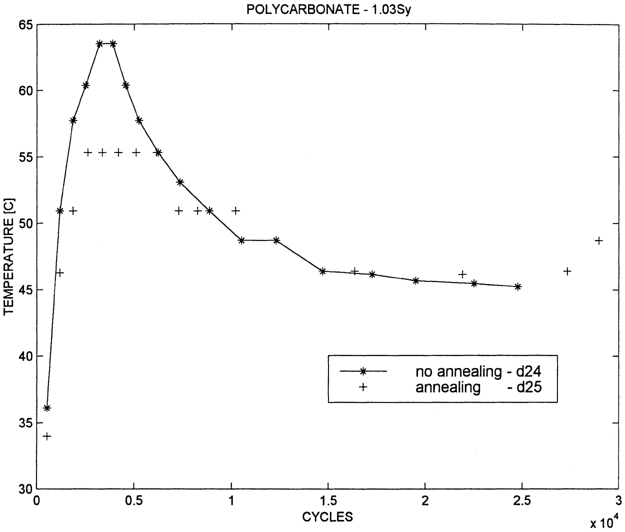

Fig. 15. Cyclic loading of two PC disks. Evolution of the core temperature. One disk was tested as-manufactured and the other was annealed prior to testing. The heat treatment affected the definition of the thermal peak. Both specimens were tested at identical stress levels.

compressive loading has been carried out at relatively low maximal stress levels for the PMMA specimens. By contrast, the PC could be cycled at a maximal stress level of the order of its yield strength without immediate collapse. For PMMA, our observations corroborated previous observations about hysteretic heating and the influence of the test frequency. For PC, a previously unreported thermal peak was observed to develop during the initial stages of the loading. It was

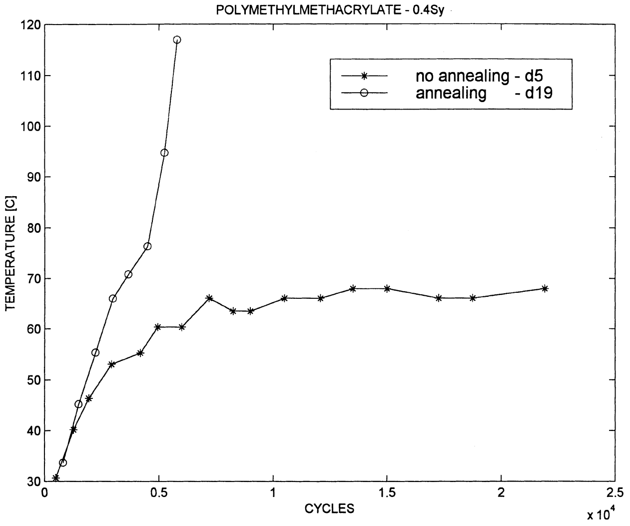

Fig. 16. Cyclic loading of two PMMA disks. Evolution of the core temperature. One disk was tested as-manufactured and the other was annealed prior to testing. The heat treatment dramatically reduces the life of the specimen. The latter responds as if it were tested to an overall higher than nominal maximum stress level.

observed that the peak gets increasingly sharper and higher with increasing relative stress.

The exact physical mechanism underlying the peak formation has not been elucidated at this stage. However, we observed that the definition of the peak (sharpness and height) is significantly reduced by annealing heat treatments. We did not observe a noticeable change in the *monotonic* mechanical or thermal behavior of the two polymers as a result of the thermal treatment. It can however be argued that for PC, since the thermal response is milder for a heat-treated material, this implies that a greater part of the mechanical energy is stored in microstructural changes during the initial phase of the test. An example of microstructural changes could be the formation of protocrazes, as in Li et al. (1995). For the PMMA specimens, the heat treatment shortened considerably the life of the specimen. Such response could be attributed to testing at higher relative stress, corresponding to a slight decrease of the yield strength of the material which was not pinpointed in the present work.

The importance of the heat transfer process was characterized by noting that the specimen's temperature is not homogeneous. Higher temperatures were recorded in the core of the specimen, when compared with its extremities. This observation is important, not only for modeling purposes but also to assess the possibility of a *localization of the temperature rise*. The thermal localization promotes a localized, bulging, failure mode in PMMA, by contrast with the diffused barreling mode observed in PC. The distinction between the diffuse and localized failure modes can be related to the homologous temperature *level* reached in the center of the specimen. Consequently, it appears that beyond the physical evidence of hysteretic heating, both the thermal boundary conditions and the thermomechanical properties of the specific material play a dominant role in the failure process.

In the next paper, we will address specific aspects of the conversion of mechanical energy into heat, with emphasis on the temperature distribution and the influence of the boundary conditions.

6. Conclusions

- A comparative study of hysteretic heating was carried out in commercial PMMA and PC specimens subjected to cyclic compressive stresses.

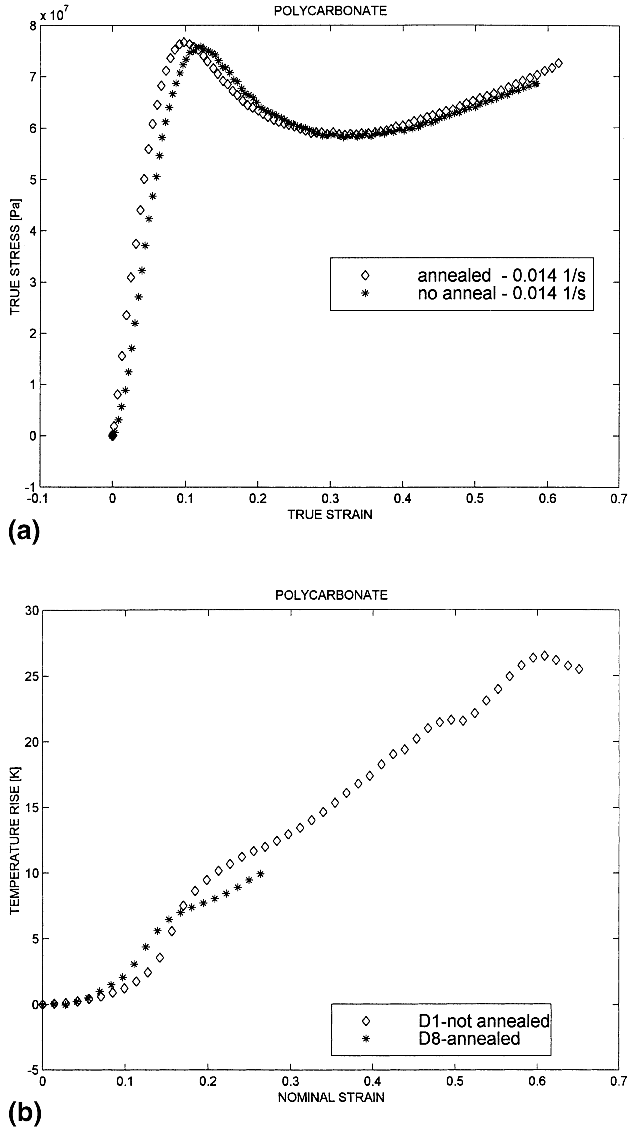

Fig. 17. (a) Monotonic loading of heat-treated PC disks. The heat treatment does not seem to affect the mechanical response of the material. (b) The corresponding evolution of the core temperature. Here too, the heat treatment does not seem to affect the thermal response of the material.

- Both materials heat up upon cycling with a pronounced influence of the cycling frequency and the applied stress level.
- Commercial PMMA is very sensitive to minor variations in the applied stress. This material fails in a very short time when the stress reaches some 0.45 of the yield strength of this material. Failure is sudden and consists of localized bulging in the central part of the specimen. Annealing heat treatments shorten the fatigue life of this material. This may result from a slight decrease of the yield strength of the material so that the effective relative stress may be higher than deemed.

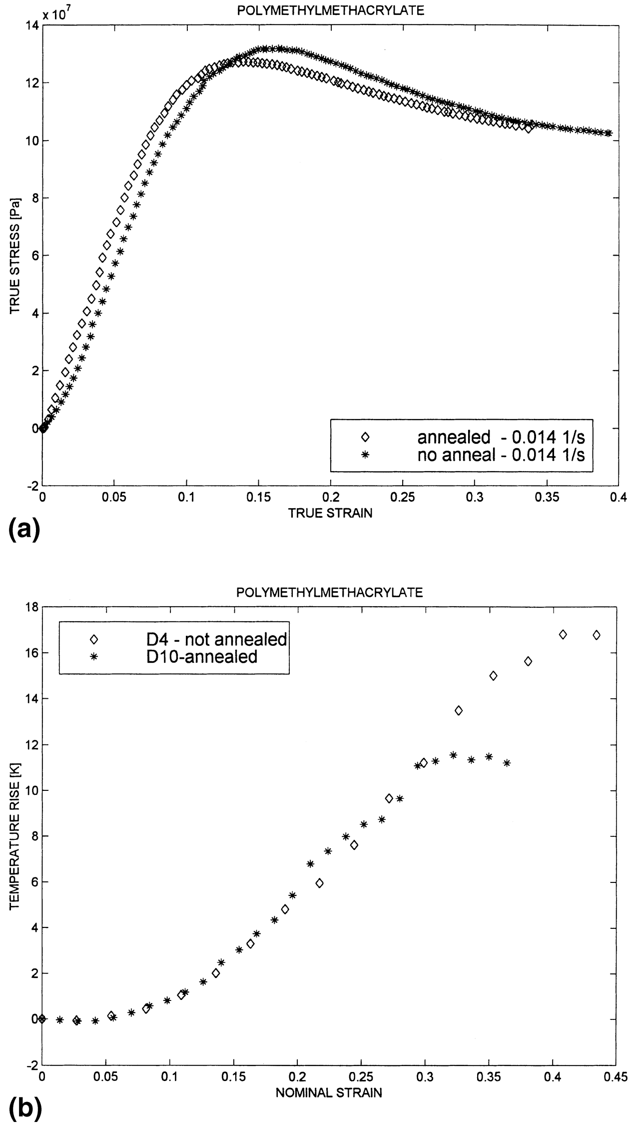

Fig. 18. (a) Monotonic loading of heat-treated PMMA disks. The heat treatment does not seem to affect the mechanical response of the material. (b) The corresponding evolution of the core temperature. Here too, the heat treatment does not seem to affect the thermal response of the material.

- Commercial PC can be tested at very high relative stress levels, of the order of the yield strength and above. For this material, a well-defined temperature peak develops during the initial stage of the loading. The definition of the peak improves with increasing stress levels and testing frequency. Failure of the specimen occurs by diffuse barreling. Thermal treatments reduce the definition of the thermal peak.
- Both materials do not exhibit a marked sensitivity in their monotonic thermal and mechanical response to the annealing treatments.
- The temperature distribution is not uniform in the specimen. Consequently, beyond the mere characterization of the hysteretic heating effect,

care should be paid both to the thermal boundary conditions of the problem and to the thermomechanical characteristics of the investigated material.

- Both the localization and the extent of the temperature rise determine the nature of the failure mechanism in the investigated materials.

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