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Technical note: Determination of the Johnson-Cook material parameters using the SCS specimen

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Abstract

This note addresses the determination of the Johnson Cook material parameters using the shear compression specimen (SCS). This includes the identification of the thermal softening effect in quasi static and dynamic loading as well as and the strain rate hardening effect in dynamic loading. A hybrid experimental-numerical (finite element) procedure is presented to identify the constitutive parameters, with an application to Ti6Al4V alloy. The present results demonstrate the suitability of the SCS for constitutive testing.

Keywords: A. shear compression specimen. B. Johnson-Cook model .C. finite elements. D. dynamic loading E. quasi-static loading

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1. Introduction

The shear compression specimen (SCS), developed by Rittel et al. (2002)[1,2], was designed to investigate the mechanical behavior of materials at large strains, over a wide range of strain-rates and temperatures. A thorough numerical investigation of the SCS for both quasi-static and dynamic loading conditions, was presented in which the material model was assumed to be bi-linear[3,4]. The validity of the specimen was established by comparing stress-strain data obtained with the SCS to those obtained from uniaxial (tension/compression) tests[2, 5]. More recently[6], these results were extended to *parabolic hardening* materials, which are quite common among metallic materials.

Several constitutive material models, which should adequately represent large strain behavior over a wide range of strain rates and temperatures, have been proposed. Examples of such models are Johnson-Cook[7] (JC), Zerilli-Armstrong[8] and Bodner-Partom[9] . The empirical model JC is widely used and incorporated in most commercial finite elements packages, and will therefore be the subject of this note.

Our purpose is to present a simple method for the determination of the JC material parameters using the SCS specimen test. The method is illustrated for a commercial Ti6Al4V alloy.

The first part, which follows this introduction, introduces the specimen geometry and material, the JC material model, and briefly reminds the data reduction technique for parabolic hardening materials under quasi-static loading[6]. Next, we introduce the technique used to determine the parameters related to thermal softening and strain rate hardening. The second part addresses the determination of the parameters of the JC model for Ti6Al4V SCS specimens. The results are discussed and compared to those available in the literature

2. Experimental and constitutive model

2.1 Specimen geometry and material data

The shear compression specimen (SCS) is shown in Fig. 1. The specimen promotes shear deformation in an inclined gauge-section, and it is aimed at large-strain constitutive testing of materials under both quasi-static and dynamic loading conditions[1-6, 10-11]. For all the investigated cases, the length, diameter, gauge thickness, gauge height and fillet root -

radius were chosen to be: $\mathrm{L} = 20\ \mathrm{mm}$, $\mathrm{D} = 10\ \mathrm{mm}$, $\mathrm{t} = 2.5\ \mathrm{mm}$, $w = 2.0\ mm$ and $\rho = 0.25\,\mathrm{mm}$ respectively. This set of geometrical parameters is commonly used in experiments

The material of this study is annealed Ti6Al4V, supplied as 12.7 mm diameter rods. The physical properties of Ti6Al4V alloy are listed in Table 1.

2.2 Johnson-Cook model and data reduction

The inelastic behavior of the investigated alloy is assumed to be described by Johnson-Cook model[7]. This material model is particularly suited to model high strain rate deformation of metals. It is generally used in adiabatic transient dynamic analysis. The hardening is a particular type of isotropic hardening in which the yield stress σ^0 is assumed to be of the form:

$$\sigma^0 = \left(A + B\left(\varepsilon^p\right)^{\mathrm{n}}\right)\left(1 + C\,\log\left(\frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}\right)\right)\left(1 - \hat{T}^{\mathrm{m}}\right) \tag{1}$$

Where

$$\hat{\mathrm{T}} = \begin{cases} 0 & for \quad \mathrm{T} < \mathrm{T_r} \\ \dfrac{\mathrm{T} - \mathrm{T_r}}{\mathrm{T_m} - \mathrm{T_r}} & for \quad \mathrm{T_r} \le \mathrm{T} \le \mathrm{T_m} \\ 1 & for \quad \mathrm{T} > \mathrm{T_m} \end{cases} \quad . \tag{2}$$

In equation (1) ε^p is the equivalent plastic strain and *A, B, C, n* and *m* are material parameters, to be identified. The natural logarithm is denoted "log". $\hat{\mathrm{T}}$ is a dimensionless temperature defined in equation (2), where T is the current temperature, $\mathrm{T_m}$ is the melting temperature and $\mathrm{T_r}$ is a reference temperature. A is the yield stress σ_{Y} at temperatures below T_r . $\dot{\varepsilon}_0$ and C are usually measured at or below the reference temperature.

The applied displacement d and the resultant force P are reduced into equivalent stress and strain according to:

$$\hat{\sigma} = \mathrm{k}_1\left(1-\mathrm{k}_2\,\hat{\varepsilon}\right)\frac{P}{D\,t} \tag{3}$$

$$\hat{\varepsilon} = \mathrm{k}_3\,\frac{d-d_Y}{h} \tag{4}$$

The coefficients for the data reduction $\left(k_1,k_2,k_3\right)$ have been previously determined for this material[10] : $k_1 = 0.96,\ k_2 = 0.18$ and $k_3 = 1.133$.

3. The method

The method is of a hybrid experimental-numerical type which requires at least three kinds of experiments in compression loading:

a. Quasi-static testing at room temperature.

b. Quasi-static testing at a higher temperature.

c. Dynamic testing at room temperature.

A preliminary stage consists of determining the k_i coefficients[6, 10].

3.1 Determination of B and n in the JC model

The experimental load-displacement curves (type a) together with the k_i coefficients and Eqns. (3)-(4) can be used to plot the plastic characteristic curve of the material at room temperature $\sigma - \varepsilon^p$. This curve is best fitted by to $\sigma^0 = A + B\left(\varepsilon^p\right)^{\mathrm{n}}$ where $A = \sigma_Y$ is known.

3.2 Determination of m in the JC model

Quasi static experimental results at both room (type a) and higher temperatures (type b) are needed to determine *m*.

If quasi static experiments, at the same $\dot{\varepsilon}^p$, are carried out at two different temperatures denoted by the superscripts (1) and (2), the ratio R between the stresses at a specific plastic strain ε_*^p can be expressed as:

$$R = \frac{\sigma_0^{(1)}\left(\varepsilon_*^p, \dot{\varepsilon}_*^p\right)}{\sigma_0^{(2)}\left(\varepsilon_*^p, \dot{\varepsilon}_*^p\right)} = \frac{1-\left(\hat{T}^{(1)}\right)^m}{1-\left(\hat{T}^{(2)}\right)^m} \tag{5}$$

If $T^{(2)} = T_r$ (room temperature), then $\hat{T}^{(2)} = 0$ (Eqn. (2)) And m is given by

$$m = \frac{\log(1-R)}{\log\left(\hat{T}^{(1)}\right)} \tag{6}$$

The method can be summarized as follows:

1. Use Eqns (3)-(4) to plot $\hat{\sigma} - \hat{\varepsilon}^p$ for experiments at room temperature and at a higher temperature.
2. Determine the ratio R using Eqn. (5) over a wide range of $\hat{\varepsilon}^p$.
3. Determine m according to Eqn. (6)

3.3 Determination of C in the JC model

Dynamic stress-strain curves, using for example a Split Hopkinson Pressure Bar (SHPB[11]) at room temperature, are needed to determine the coefficient C which corresponds to the strain rate effect. The experimental load displacement curve should be reduced for SCS specimens into a strain-stress curve with the aid of Eqns (3)-(4). This curve is then best fitted to Eq. (1) using the previously obtained parameters, with only one unknown, namely the parameter C. The obtained value of C is then verified numerically.

4. Example

4.1 Determination of B and n

The quasi-static line for 24°C in figure 2 was best fitted using least square to the formula: $\sigma_0\left[MPa\right] = 880 + B\left(\varepsilon^p\right)^n$. The obtained parameters with R^2: 0.9923 are $B = 695\ MPa$ and $n = 0.3582$.

4.2 Determination of m

The stresses shown in Fig. 2a for temperatures 213°C and 416°C are divided by the stresses at 24°C according to Eqn. (5). The result is plotted *vs.* ε^p in Fig. 2b as well as the average values in the range $0.05 < \varepsilon^p < 0.35$. The averaged values are R=0.8061, and R=0.6984 for 213°C and 416°C respectively. Substituting these values into Eqn . (6) results in m = 0.7601 for 213°C and m = 0.8388 for 416°C. An average value of m = 0.8 is adopted.

4.3 Determination of C

A first estimation for the coefficient C is done by best fitting the dynamic experimental stress-strain curves to the JC model with the already obtained material parameters. It is assumed that the reference strain rate of Eqn. (1) is $\hat{\varepsilon}_0 = 1\ 1/s$. It is further assumed that the specimen remains at room temperature, thus neglecting thermo-plastic coupling effects, and hence $\hat{T} = 0$. The average strain rate was 3000 1/s, and it was assumed that this strain rate is constant during the experiment.

Under these assumptions the experimental stress – plastic strain curves shown in figure 3 were fitted with the formula:

$$\sigma^0\left[MPa\right] = \left(880 + 695\left(\varepsilon^p\right)^{0.3582}\right)\left(1 + C\log\left(\frac{3000}{1}\right)\right) \tag{7}$$

For experiment 1 the obtained value is C = 0.041 while for experiment 2 the obtained value is C = 0.051. Even for two experiments carried out at the same strain rate, a difference of 0.01 in the value of C (equivalent to 25%), is observed. Increasing the number of experiments at different strain rates will certainly improve the statistical reliability of the obtained value of C over a wider range of the dynamic strain rate region. In the present paper, we explain how to get this value using a minimum number of experiments. It is therefore expected that the best fit here will be obtained at high strain rates close to 3000 1/s at which the experiments were conducted.

The obtained values of C were verified numerically using the commercial finite elements code Abaqus explicit version 6.7. The material model was JC with the above reported parameters, together with a representative thermomechanical conversion factor $\beta = 0.4$ according to the results of Rittel and Wang[12]. The properties of table 1 were used as well. The experimentally measured velocities on the top and bottom face of the specimen were applied as boundary conditions. The reaction force on the bottom of the specimen (adjacent to the transmission bar of the SHPB) was determined for different values of C namely: 0.02, 0.04 and 0.06. Figure 4 shows the numerical obtained reaction for the three values of C together with the experimental measured values. A very good agreement is observed for C = 0.04.

5. Summary and conclusions

Assuming that Ti6Al4V obeys the Johnson-Cook material model, its parameters were determined using the SCS specimen . As a preliminary remark, it should be noted that the JC model is a linear function of the logarithm of the normalized strain rate. Such a linear relation is seldom observed for most metals and alloys. Usually two regions are evident[13], with a transition between them at strain rates in the range of $10^2 - 10^3\ \mathrm{s}^{-1}$. Consequently, a more accurate representation of the material behavior would require a reference strain rate of the order of the transition strain rate. However, this work, and many others assume a reference strain-rate of $\dot{\varepsilon} = 1\,\mathrm{s}^{-1}$, thus extending the range of validity of the JC model into the quasi-static regime. We adopted the "traditional" value of $\dot{\varepsilon} = 1\,\mathrm{s}^{-1}$ to allow for comparison with previous work.

An experimental-numerical approach is used. Three types of experiments were used: (1) Quasi-static testing at room temperature. (2) Quasi-static testing at a higher temperature.

(3) Dynamic testing at room temperature. The numerical simulations are first used to determine the data reduction coefficients $\left(k_1, k_2, k_3\right)$ and further used to verify the value of C. The determined parameters are detailed in table 2 and compared to those obtained by other investigations[14-16]. An overall very good agreement is observed, with the exception that the current work predicts that the material is more strain rate sensitive since the value of C is almost three times the value obtained in other investigations[15, 16].It is noted that the methodology described here may also be used with other specimens (cylinders for example), with adequate data reduction procedures. In all cases, the value of the C parameter can be fine tuned using numerical simulations, as illustrated for the SCS specimen.

Acknowledgment

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Captions of figures

Fig. 1 - shear compression specimen.

Fig. 2 - Quasi static experimental results. a. The quasi-static experimental characteristic curves at temperatures: 24^{o}C, 213^{o}C and 416^{o}C . Note the thermal softening effect. b. The ratio between stresses at elevated temperature (213^{o}C and 416^{o}C) to stresses at room temperature (24^{o}C) and their average value R (Eqn. (5)).

Fig. 3 - The dynamic obtained stress-plastic strain curves together with the fitted JC model of Eqn. (7) with the obtained parameters C = 0.041 and 0.051. . The averaged strain rate is 3000 1/s.

Fig. 4 - The numerical calculated reaction force vs. time on the bottom face of the specimen in the SHPB machine for C = 0.02, 0.04 and 0.06 along with the two experimental results. Note the good agreement for C=0.04. . The averaged strain rate is 3000 1/s.

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FIGURES

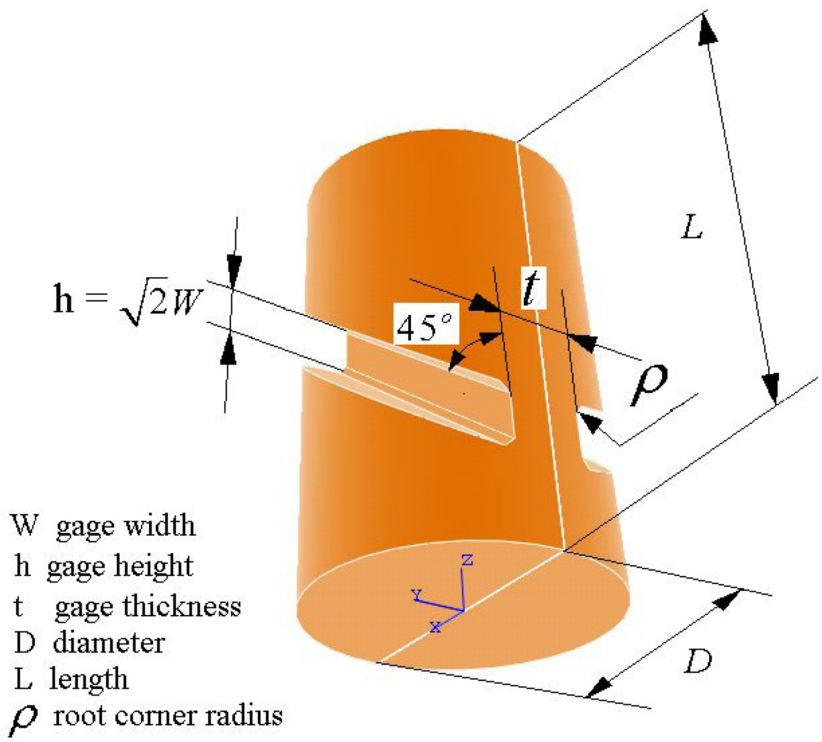
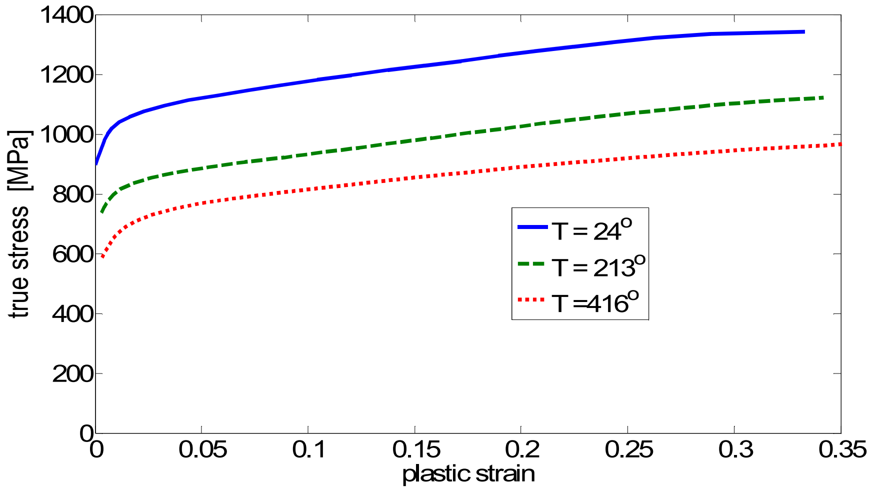
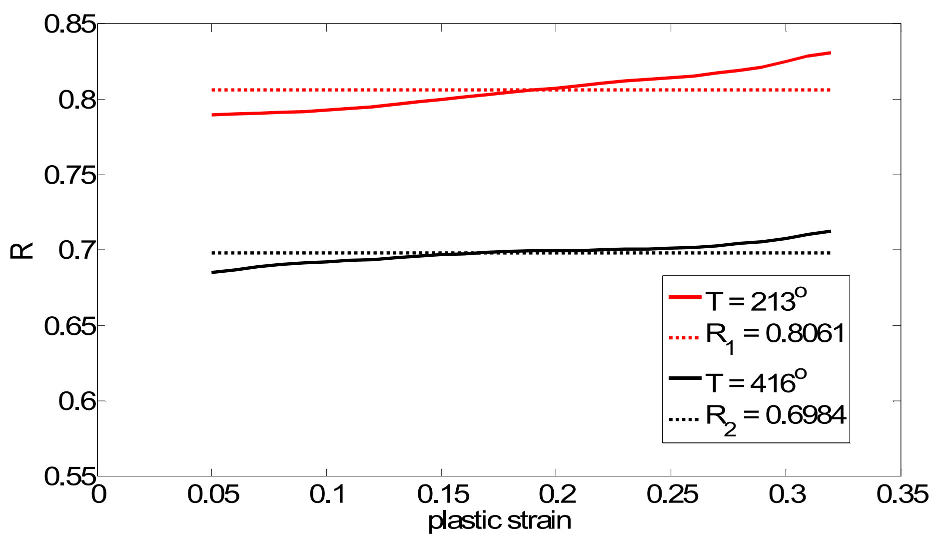


Fig. 1- The shear compression specimen

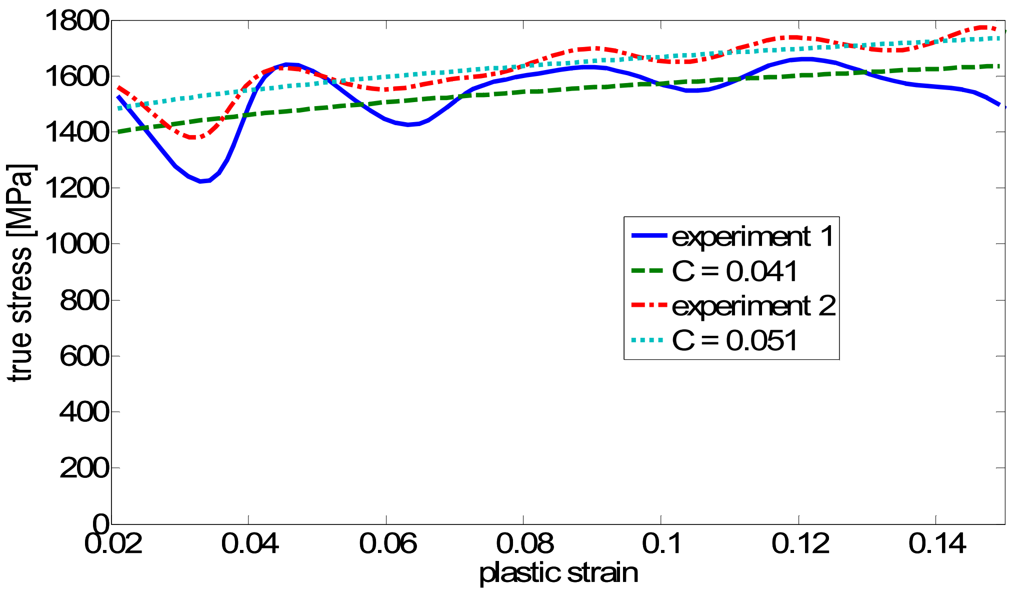


a.



b

Fig. 2 - Quasi static experimental results. a. The quasi-static experimental characteristic curves at temperatures: 24°C, 213°C and 416°C . Note the thermal softening effect. b. The ratio between stresses at elevated temperature (213°C and 416°C) to stresses at room temperature (24°C) and their average value R (Eqn. (5)).



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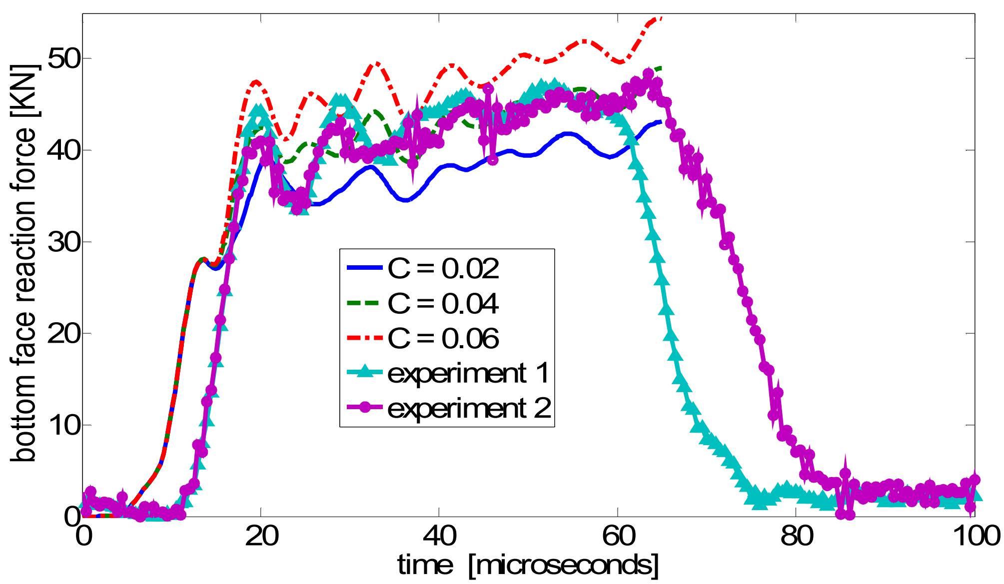

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TABLES

Table 1: properties of Ti6Al4V

Material	$E\ [GPa]$	ν	$\sigma_Y\ [MPa]$	$\rho\ [Kg/m^3]$	$c_p\ [J/(Kg\ C^\circ)]$	$T_m\ [C^\circ]$
Ti6Al4V	113.8	0.342	880	4430	526.3	1660

Table 2: A comparison of reported values of the Johnson-Cook parameters

	A [MPa]	B [MPa]	n	m	C	$\dot{\varepsilon}_0$
This work	880	695	0.36	0.8	0.04	1
Seo et al (2005)	998	653	0.45	0.7	0.0198	1
Meyer and Kleponis (2001)	896	656	0.5	0.8	0.0128	1
Meyer and Kleponis (2001) CTH library standard material	863	331	0.34	0.8	0.0120	1
Lee and Lin (1998)	724	683	0.47	1	0.035	10^{-5}